

Package ‘spsur’

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Type Package

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Description A collection of functions to test and estimate Seemingly Unrelated Regression (usually called SUR) models, with spatial structure, by maximum likelihood and three-stage least squares. The package estimates the most common spatial specifications, that is, SUR with Spatial Lag of X regressors (called SUR-SLX), SUR with Spatial Lag Model (called SUR-SLM), SUR with Spatial Error Model (called SUR-SEM), SUR with Spatial Durbin Model (called SUR-SDM), SUR with Spatial Durbin Error Model (called SUR-SDEM), SUR with Spatial Autoregressive terms and Spatial Autoregressive Disturbances (called SUR-SARAR) and SUR with Spatially Independent Model (called SUR-SIM).

The methodology of these models can be found in next references

Mur, J., Lopez, F., and Herrera, M. (2010) <doi:10.1080/17421772.2010.516443>

Lopez, F.A., Mur, J., and Angulo, A. (2014) <doi:10.1007/s00168-014-0624-2>.

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R topics documented:

spsur-package	2
dgp_spsur	6
impacts	10
lmtestspsur	14
lrtestspsur	17
lr_betas_spsur	21
NCOVR	24
print.summary.spsur	26
spc	26
spsur3sls	27
spsurml	31
spsurtime	38
summary.spsur	42
W	43
wald_betas	43
wald_deltas	45
Wspc	48
Index	49

spsur-package	<i>Spatial Seemingly Unrelated Regression Models.</i>
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Description

spsur offers the user a collection of functions to estimate Spatial Seemingly Unrelated Regression (SUR) models by maximum likelihood or three-stage least squares, using spatial instrumental variables. Moreover, **spsur** obtains a collection of misspecification tests for omitted or wrongly specified spatial structure. The user will find spatial models more popular in applied research such as the SUR-SLX, SUR-SLM, SUR-SEM, SUR-SDM, SUR-SDEM and SUR-SARAR, plus the spatially independent SUR, or SUR-SIM.

Details

Some functionalities that have been included in **spsur** package are:

1. Testing for spatial effects

The function `lmtestspsur` provides a collection of Lagrange Multipliers, LM, for testing different forms of spatial dependence in SUR models. They are extended versions of the well-known LM tests for omitted lags of the explained variable in the right hand side of the equation, LM-SLM, the LM tests for omitted spatial errors, LM-SEM, the join test of omitted spatial lags and spatial errors, LM-SARAR, and the robust version of the first two Lagrange Multipliers, LM*-SLM and LM*-SEM.

These tests can be applied to models always with a SUR nature. Roughly, we may distinguish two situations:

- Datasets with a single equation $G=1$, for different time periods $Tm>1$ and a certain number of spatial units in the cross-sectional dimension, N . This is what we call *spatial panel datasets*. In this case, the SUR structure appears in form of (intra) serial dependence in the errors of each spatial unit.
- Datasets with a several equations $G>1$, different time periods $Tm>1$ and a certain number of spatial units, N . The SUR structure appears, as usual, because the errors of the spatial units for different equations are contemporaneously correlated.

2. Estimation of the Spatial SUR models

As indicated above, `spsur` package may work with a list of different spatial specifications. They are the following:

- *SUR-SIM*: SUR model without spatial effects

$$y_{tg} = X_{tg}\beta_g + \epsilon_{tg}$$

- *SUR-SLX*: SUR model with spatial lags of the regressors

$$y_{tg} = X_{tg}\beta_g + W X_{tg}\theta_g + \epsilon_{tg}$$

- *SUR-SLM*: SUR model with spatial lags of the endogenous

$$y_{tg} = \lambda_g W y_{tg} + X_{tg}\beta_g + \epsilon_{tg}$$

- *SUR-SEM*: SUR model with spatial errors

$$y_{tg} = X_{tg}\beta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

- *SUR-SDM*: SUR model with spatial lags of the endogenous variable and of the regressors or Spatial Durbin model

$$y_{tg} = \lambda_g W y_{tg} + X_{tg}\beta_g + W X_{tg}\theta_g + \epsilon_{tg}$$

- *SUR-SDEM*: SUR model with spatial errors and spatial lags of the endogenous variable and of the regressors

$$y_{tg} = X_{tg}\beta_g + W X_{tg}\theta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

- *SUR-SARAR*: Spatial lag model with spatial errors

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

where y_{tg} , u_{tg} and ϵ_{tg} are $(Nx1)$ vectors; X_{tg} is a matrix of regressors of order (NxP) ; λ_g and ρ_g are parameters of spatial dependence and W is the (NxN) spatial weighting matrix.

These specifications can be estimated by maximum-likelihood methods, using the function `spsurml`. Moreover, the models that include spatial lags of the explained variables in the right hand side of the equations, and the errors are assumed to be spatially uncorrelated (namely, the SUR-SLM and the SUR-SDM), can also be estimated using three-stage least-squares, `spsur3sls`, using spatial instrumental variable to correct for the problem of endogeneity present in these cases.

3. Diagnostic tests

Testing for inconsistencies or misspecifications in the results of an estimated (SUR) model should be a primary task for the user. `spsur` focuses, specifically, on two main question such as omitted or wrongly specified spatial structure and the existence of structural breaks or relevant restrictions in the parameters of the model. In this sense, the user will find:

1. *Marginal tests*

The Marginal Multipliers test for omitted or wrongly specified spatial structure in the equations. They are routinely part of the output of the maximum-likelihood estimation, shown by `spsurml`. In particular, the $LM(\lambda|\rho)$ tests for omitted spatial lags in a model specified with spatial errors (SUR-SEM; SUR-SDM). The $LM(\rho|\lambda)$ tests for omitted spatial error in a model specified with spatial lags of the explained variable (SUR-SLM; SUR-SDM).

2. *Coefficients stability tests*

`spsur` includes two functions designed to test for linear restrictions on the β coefficients of the models and on the spatial coefficients (λ s and ρ s terms). The function for the first case is `wald_betas` and `wald_deltas` that of the second case. The user has ample flexibility to define different forms of linear restrictions, so that it is possible, for example, to test for their time constancy to identify structural breaks.

4. Marginal effects

In recent years, since the publication of LeSage and Pace (2009), it has become popular in spatial econometrics to evaluate the multiplier effects that a change in the value of a regressor, in a point in the space, has on the explained variable. `spsur` includes a function, `impacts`, that computes these effects. Specifically, `impacts` obtains the average, over the N spatial units and Tm time periods, of such a change on the contemporaneous value of the explained variable located in the same point as the modified variable. This is the so-called *Average Direct effect*. The *Average Indirect effect* measure the proportion of the impact that spills-over to other locations. The sum of the two effects is the *Average Total effect*.

These estimates are complemented with a measure of statistical significance, following the randomization approach suggested by LeSage and Pace (2009).

5. Additional functionalities

A particular feature of **spsur** is that the package allows to obtain simulated datasets with a SUR nature and the spatial structure decided by the user. This is the purpose of the function `dgp_spsur`. The function can be inserted into a more general code to solve, for example, Monte Carlo studies related to these type of models or, simply, to show some of the stylized characteristics of a SUR model with certain spatial structure.

Datasets

spsur includes four different datasets: `spc`, `NCOVR`, Italian unemployment and simulated dataset. The four sets are used to illustrate the capabilities of different functions. Briefly, their main characteristics are the following

- The `spc` dataset (Spatial Phillips-Curve) is a classical dataset taken from Anselin (1988, p. 203), of small dimensions.
- The `NCOVR` dataset comprises Homicides and a list of selected socio-economic variables for continental U.S. counties in four decennial census years: 1960, 1970, 1980 and 1990. It is freely available from <https://geodacenter.github.io/data-and-lab/ncovr/>. `NCOVR` is a typical spatial panel dataset ($G=I$).
- `Italian unemployment` which includes a panel data of unemployment in Italian provinces for the period 1996-2014. Once again, it is a spatial panel dataset ($G=I$)
- A data set `Slm`, obtained using the function `dgp_spsur`, to show how to input data using matrices, as an alternative to the use of `Formula` and `data.frame`.

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References

- Breusch T, Pagan A (1980). The Lagrange multiplier test and its applications to model specification in econometrics. *Review of Economic Studies* 47: 239-254.
- LeSage, J., and Pace, R. K. (2009). *Introduction to spatial econometrics*. Chapman and Hall/CRC.
- Lopez, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.
- López, F.A., Martínez-Ortiz, P.J., and Cegarra-Navarro, J.G. (2017). Spatial spillovers in public expenditure on a municipal level in Spain. *Annals of Regional Science*, 58(1), 39-65.

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.

dgp_spsur

Generation of a random dataset with a spatial SUR structure.

Description

The purpose of the function `dgp_spsur` is to generate a random dataset with the dimensions and spatial structure decided by the user. This function may be useful in pure simulation experiments or with the aim of showing specific properties and characteristics of a spatial SUR dataset and inferential procedures related to them.

The user of `dgp_spsur` should think in terms of a Monte Carlo experiment. The arguments of the function specify the dimensions of the dataset to be generated, the spatial mechanism underlying the data, the intensity of the SUR structure among the equations and the values of the parameters to be used to obtain the simulated data, which includes the error terms, the regressors and the explained variables.

```
dgp_spsur <- function(Sigma, Tm = 1, G, N, Betas, Thetas = NULL, durbin = FALSE, rho = NULL,
lambda = NULL, p = NULL, W = NULL, X = NULL, pdfU = "nvrnorm", pdfX = "nvrnorm")
```

Usage

```
dgp_spsur(Sigma, Tm = 1, G, N, Betas, Thetas = NULL, rho = NULL,
lambda = NULL, p = NULL, W = NULL, X = NULL, pdfU = "nvrnorm",
pdfX = "nvrnorm")
```

Arguments

<code>Sigma</code>	Covariance matrix between the G equations of the SUR model. This matrix should be definite positive and the user must check for that.
<code>Tm</code>	Number of time periods. Default = 1
<code>G</code>	Number of equations.
<code>N</code>	Number of cross-section or spatial units
<code>Betas</code>	A row vector of order $(1 \times P)$ showing the values for the <i>beta</i> coefficients. The first P_1 terms correspond to the first equation (where the first element is the intercept), the second P_2 terms to the coefficients of the second equation and so on.
<code>Thetas</code>	Values for the θ coefficients in the G equations of the model, when the type of spatial SUR model to be simulated is a " slx ", " sdm " or " sdem ". <i>Thetas</i> is a row vector of order $1 \times P \times Theta$, where $P \times Theta = p - G$; let us note that the intercept cannot appear among the spatial lags of the regressors. The first $1 \times K \times Theta_1$ terms correspond to the first equation, the second $1 \times P \times Theta_2$ terms correspond to the second equation, and so on. Default = NULL.

rho	Values of the coefficients $\rho_g; g = 1, 2, \dots, G$ related to the spatial lag of the errors in the G equations. If <i>rho</i> is an scalar and there are G equations in the model, the same value will be used for all the equations. If <i>rho</i> is a row vector, of order $(1 \times G)$, the function <code>dgp_spsur</code> will use these values, one for each equation of the spatial errors. Default = NULL.
lambda	Values of the coefficients $\lambda_g; g = 1, 2, \dots, G$ related to the spatial lag of the explained variable of the g -th equation. If <i>lambda</i> is an scalar and there are G equations in the model, the same value will be used for all the equations. If <i>lambda</i> is a row vector, of order $(1 \times G)$, the function <code>dgp_spsur</code> will use these values, one for each equation. Default = NULL.
p	Number of regressors by equation, including the intercept. <i>p</i> can be a row vector of order $(1 \times G)$, if the number of regressors is not the same for all the equations, or a scalar, if the G equations have the same number of regressors.
W	A spatial weighting matrix of order $(N \times N)$, assumed to be the same for all equations and time periods.
X	This argument tells the function <code>dgp_spsur</code> which X matrix should be used to generate the SUR dataset. If X is different from NULL, <code>{dgp_spsur}</code> will upload the X matrix selected in this argument. Note that the X must be consistent with the dimensions of the model. If X is NULL, <code>dgp_spsur</code> will generate the desired matrix of regressors from a multivariate Normal distribution with mean value zero and identity $(P \times P)$ covariance matrix. As an alternative, the user may change this probability distribution function to the uniform case, $U(0, 1)$, through the argument pdfX . Default = NULL.
pdfU	Multivariate probability distribution function, mdf, from which the values of the error terms will be drawn. The covariance matrix is the Σ matrix specified by the user in the argument Sigma . The function <code>dgp_spsur</code> provides two mdf, the multivariate Normal, which is the default, and the log-Normal distribution function which means just exponentiate the sampling drawn from a $N(0, \Sigma)$ distribution. Default = "nvrnorm".
pdfX	Multivariate probability distribution function, mdf, from which the values of the regressors will be drawn. The regressors are assumed to be independent. <code>dgp_spsur</code> provides two mdf, the multivariate Normal, which is the default, and the uniform in the interval $U[0, 1]$, using the <code>dunif</code> function, <code>dunif</code> , from the stats package. Default = "nvrnorm".

Details

The purpose of the function `dgp_spsur` is to generate random datasets, of a SUR nature, with the spatial structure decided by the user. The function requires certain information to be supplied externally because, in fact, `dgp_spsur` constitutes a Data Generation Process, DGP. The following aspects should be addressed:

- The user must define the dimensions of the dataset, that is, number of equations, G , number of time periods, Tm , and number of cross-sectional units, N .
- Then, the user must choose the type of spatial structure desired for the model from among the list of candidates of "**sim**", "**slx**", "**slm**", "**sem**", "**sdm**", "**sdem**" or "**sarar**"; the default is the "**sim**" specification which does not have spatial structure. The decision is made implicitly,

just omitting the specification of the spatial parameters which are not involved in the model (i.e., in a "slm" there are no ρ parameters but appear λ parameters; in a "sdem" model there are ρ and θ parameters but no λ coefficients). Of course, if the user needs a model with spatial structure, a $(n \times N)$ weighting matrix, W , should be chosen.

- The next step builds the equations of the SUR model. In this case, the user must specify the number of regressors that intervene in each equation and the coefficients, β parameters, associated with each regressor. The *first* question is solved through the argument p which, if a scalar, indicates that the same number of regressors should appear in all the equations of the model; if the user seeks for a model with different number of regressors in the G equations, the argument p must be a $(1 \times G)$ row vector with the required information. It must be remembered that dgp_spsur assumes that an intercept appears in all equations of the model.

The *second* part of the problem posited above is solved through the argument $Betas$, which is a row vector of order $(1 \times p)$ with the information required for this set of coefficients.

- The user must specify, also, the values of the spatial parameters corresponding to the chosen specification; we are referring to the λ_g , ρ_g and θ_g , for $g = 1, \dots, G$ and $k = 1, \dots, K_g$ parameters. This is done through the arguments $lambda$, rho and $theta$. The first two, $lambda$ and rho , work as K : if they are scalar, the same value will be used in the G equations of the SUR model; if they are $(1 \times G)$ row vectors, a different value will be assigned for each equation.

Moreover, $theta$ works like the argument $beta$. The user must define a row vector of order $1 \times P$ $theta$ showing these values. It is worth to remember that in no case the intercept will appear among the lagged regressors.

- Finally, the user must decide which values of the regressors and of the error terms are to be used in the simulation. The regressors can be uploaded from an external matrix generated previously by the user. This is the argument X . It is the responsibility of the user to check that the dimensions of the external matrix are consistent with the dataset required for the SUR model. A second possibility implies the regressors to be generated randomly by the function [dgp_spsur](#). In this case, the user must select the probability distribution function from which the corresponding data (of the regressors and the error terms) are to be drawn.

dgp_spsur provides two multivariate distribution functions, namely, the Normal and the log-Normal for the errors (the second should be taken as a clear departure from the standard assumption of normality). In both cases, random matrices of order $(Tm \times NxG)$ are obtained from a multivariate normal distribution, with a mean value of zero and the covariance matrix specified in the argument $Sigma$; then, this matrix is exponentiated for the log-Normal case. Roughly, the same procedure applies for drawing the values of the regressor. There are two distribution functions available, the normal and the uniform in the interval $U[0, 1]$; the regressors are always independent.

Value

A list with a vector Y of order $(Tm \times NG \times 1)$ with the values generated for the explained variable in the G equations of the SUR and a matrix XX of order $((Tm \times NG \times sum(p)))$, with the values generated for the regressors of the SUR, including an intercept for each equation.

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See Also

[spsurml](#), [spsur3sls](#), [spsurtime](#)

Examples

```
#####
##### CROSS SECTION DATA #####
#####

#####
### Example 1: DGP SLM model
#####
rm(list = ls()) # Clean memory
Tm <- 1 # Number of time periods
G <- 3 # Number of equations
N <- 50 # Number of spatial elements
p <- 3 # Number of independent variables
Sigma <- matrix(0.3, ncol = G, nrow = G)
diag(Sigma) <- 1
Betas <- c(1,2,3,1,-1,0.5,1,-0.5,2)
lambda <- 0.5 # level of spatial dependence
rho <- 0.0 # spatial autocorrelation error term = 0
# random coordinates
co <- cbind(runif(N,0,1),runif(N,0,1))
W <- spdep::nb2mat(spdep::knn2nb(spdep::knearneigh(co, k = 5,
longlat = FALSE)))

DGP <- dgp_spsur(Sigma = Sigma, Betas = Betas,
               rho = rho, lambda = lambda, Tm = Tm,
               G = G, N = N, p = p, W = W)
SLM <- spsur3sls(W = W, X = DGP$X, Y = DGP$Y, Tm = Tm, N = N, G = G,
               p = c(3,3,3), type = "slm")
summary(SLM)

#####
### Example 2: DGP SEM model with Tm>1; G=1 and
### different p for each equation
#####

## It usually requires 1-2 minutes maximum
rm(list = ls()) # Clean memory
Tm <- 3 # Number of time periods
G <- 1 # Number of equations
N <- 500 # Number of spatial elements
p <- c(2,3,4) # Number of independent variables
Sigma <- matrix(0.8, ncol = Tm, nrow = Tm)
```

```

diag(Sigma) <- 1
Betas <- c(1,2,1,2,3,1,2,3,4)
lambda <- 0 # level of spatial dependence = 0
rho <- c(0.2,0.5,0.8) # spatial autocorrelation error terms for each equation
# random coordinates
co <- cbind(runif(N, 0, 1),runif(N, 0, 1))
W <- spdep::nb2mat(spdep::knn2nb(spdep::knearneigh(co, k = 5,
                                                longlat = FALSE)))
DGP2 <- dgp_spsur(Sigma = Sigma, Betas = Betas, rho = rho, lambda = lambda,
                 Tm = Tm, G = G, N = N, p = p, W = W)
SLM2 <- spsurml(W = W, X = DGP2$X, Y = DGP2$Y, Tm = Tm, N = N, G = G,
              p = c(2,3,4), type = "slm")
summary(SLM2)
SEM2 <- spsurml(W = W, X = DGP2$X, Y = DGP2$Y, Tm = Tm, N = N, G = G,
              p = c(2,3,4), type = "sem")
summary(SEM2)

#####
### Example with G>1 and Tm>>1
#####
## It usually requires 1-2 minutes maximum
rm(list = ls()) # Clean memory
Tm <- 10 # Number of time periods
G <- 3 # Number of equations
N <- 100 # Number of spatial elements
p <- 3 # Number of independent variables
Sigma <- matrix(0.5, ncol = G, nrow = G)
diag(Sigma) <- 1
Betas <- rep(1:3, G)
lambda <- c(0.5, 0.1, 0.8)
rho <- 0.0 # spatial autocorrelation error term = 0
# random coordinates
co <- cbind(runif(N,0,1),runif(N,0,1))
W <- spdep::nb2mat(spdep::knn2nb(spdep::knearneigh(co, k = 5,
                                                longlat = FALSE)))
DGP3 <- dgp_spsur(Sigma = Sigma, Betas = Betas, rho = rho, lambda = lambda,
                 Tm = Tm, G = G, N = N, p = p, W = W)
SLM3 <-spsurml(Y= DGP3$Y, X = DGP3$X, G = G, N = N, Tm = Tm,
              p = p, W = W, type = "slm")
summary(SLM3)

## slm with demeaning

SLM3_dem <-spsurml(Y= DGP3$Y, X = DGP3$X, G = G, N = N, Tm = Tm,
                 p = p, W = W, type = "slm", demean = TRUE)
summary(SLM3_dem)

```

Description

This function obtains the multiplier effects, on the explained variable, of a change in a regressor for the model that has been estimated. For reasons given below, this function only applies to models with an autoregressive structure ("**slm**", "**sdm**" and "**sarar**") or with spatial lags of the regressors ("**slx**", "**sdem**").

The measurement of the multiplier effects is a bit more complicated than in a pure time series context because, due to the spatial structure of the model, part of the impacts spills over non uniformly over the space. Using the notation introduced by LeSage and Pace (2009) we distinguish between:

- **Average Direct effects:** The average over the N spatial units and Tm time periods of the effect of a unitary change in the value of a explanatory variable on the contemporaneous value of the corresponding explained variable, located in the same point of the intervened regressor. This calculus is solved for all the regressors that appear in the G equations of the model.
- **Average Indirect effects:** The average over the N spatial units and Tm time periods of the effects of a unitary change in the value of a explanatory variable on the contemporaneous value of the corresponding explained variable, located in a different spatial unit that that of the intervened regressor. This calculus is solved for all the regressors that appear in the G equations of the model.
- **Average total effects:** The sum of Direct and Indirect effects.

The information on the three estimated effects is supplement with an indirect measure of statistical significance obtained from the randomization approach introduced in LeSage and Pace (2009).

Usage

```
impacts(spsurfit, nsim = 1000)
```

Arguments

<code>spsurfit</code>	A fitted object of class <code>spsur</code> .
<code>nsim</code>	Number of simulations for the randomization procedure. Default = 1000.

Details

LeSage and Pace (2009) adapt the classical notion of '*economic multiplier*' to the problem of measuring the impact that a unitary change in the value of a regressor, produced in a certain point in space, has on the explained variable. The question is interesting because, due to the spatial structure of the model, the impacts of such change spill non uniformly over the space. In fact, the reaction of the explained variable depends on its relative location in relation to the point of intervention.

To simplify matters, LeSage and Pace (2009) propose to obtain aggregated multipliers for each regressor, just averaging the N^2 impacts that results from intervening the value of each regressor on each of the N points in Space, on the explained variable, measured also in each of the N points in space. This aggregated average is the so-called *Total effect*.

Part of this impact will be absorbed by the explained variable located in the same point of the regressor whose value has been changed (for example, the k -th regressor in the g -th equation, in the

n-th spatial unit) or, in other words, we expect that $[dy_{tgn}]/[dx_{ktgm}]ne0$. The aggregated average for the N points in space ($n=1,2,\dots,N$) and Tm time periods is the so-called *Direct effect*. The difference between the *Total effect* and the *Direct effect* measures the portion of the impact on the explained variable that leaks to other points in space, $[dy_{tgn}]/[dx_{ktgm}]for\text{nnem}$; this is the *Indirect effect*.

`impacts` obtains the three multipliers together with an indirect measure of statistical significance, according to the randomization approach described in Lesage and Pace (2009). Briefly, they suggest to obtain a sequence of $nsim$ random matrices of order $(NTmxG)$ from a multivariate normal distribution $N(0; \mathbf{Sigma})$, being \mathbf{Sigma} the estimated covariance matrix of the G equations in the SUR model. These random matrices, combined with the observed values of the regressors and the estimated values of the parameters of the corresponding spatial SUR model, are used to obtain simulated values of the explained variables. Then, for each one of the $nsim$ experiments, the SUR model is estimated, and the effects are evaluated. The function `impacts` obtains the standard deviations of the $nsim$ estimated effects in the randomization procedure, which are used to test the significance of the estimated effects for the original data.

Finally, let us note that this is a SUR model where the G equations are connected only through the error terms. This means that if we intervene a regressor in equation g , in any point in space, only the explained variable of the same equation g should react. The impacts do not spill over equations. Moreover, the impact of a regressor, intervened in the spatial unit n , will cross the borders of this spatial unit only if in the right hand side of the equation there are spatial lags of the explained variables or of the regressors. In other words, the *Indirect effect* is zero for the "**sim**" and "**sem**" models. `impacts` produces no output for these two models. Lastly, it is clear that all the impacts are contemporaneous because the equations in the SUR model have no time dynamics.

Value

Returns the Direct, Indirect and Total effects of the estimated spatial SUR model and simulated significance measures.

<code>table_dir_eff</code>	Table of average direct effects.
<code>table_ind_eff</code>	Table of average indirect effects.
<code>table_tot_eff</code>	Table of average total effects.

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References

- LeSage, J., and Pace, R. K. (2009). *Introduction to spatial econometrics*. Chapman and Hall/CRC.
- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR

framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.

See Also

[spsurml](#), [spsur3sls](#)

Examples

```
#####
### PURE CROSS SECTIONAL DATA(G>1; Tm=1) #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
## A SUR-SLM model.
spcsur.slm <-spsur3sls(Form = Tformula, data = spc, type = "slm", W = Wspc)
summary(spcsur.slm)
eff.spcsur.slm <- impacts(spcsur.slm, nsim = 30)

## Each case usually takes 1-2 minutes maximum

## A SUR-SDM model
spcsur.sdm <-spsurml(Form = Tformula, data = spc, type = "sdm", W = Wspc)
summary(spcsur.sdm)
eff.spcsur.sdm <- impacts(spcsur.sdm, nsim = 300)

## A SUR-SLX model
spcsur.slx <-spsurml(Form = Tformula, data = spc, type = "slx", W = Wspc)
summary(spcsur.slx)
eff.spcsur.slx <- impacts(spcsur.slx, nsim = 300)

## A SUR-SDM model
spcsur.sdem <-spsurml(Form = Tformula, data = spc, type = "sdem", W = Wspc)
summary(spcsur.sdem)
eff.spcsur.sdem <- impacts(spcsur.sdem, nsim = 300)

## A SUR-SARAR model
spcsur.sarar <-spsurml(Form = Tformula, data = spc, type = "sarar", W = Wspc)
summary(spcsur.sarar)
eff.spcsur.sarar <- impacts(spcsur.sarar, nsim = 300)

#####
##### G=1; Tm>1 #####
#####
```

```

### Only execute if you have enough time...
rm(list = ls()) # Clean memory
data(NCOVR)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90
## A SUR-SLM model
NCOVRSUR.slm <- spsurml(Form = Tformula, data = NCOVR, type = "slm", W = W)
summary(NCOVRSUR.slm)
eff.NCOVR.slm <- impacts(NCOVRSUR.slm, nsim = 100)

```

Imtestpsur

Testing for the presence of spatial effects in Seemingly Unrelated Regressions

Description

The function `spsurml` reports a collection of Lagrange Multipliers designed to test for the presence of different forms of spatial dependence in a *SUR* model of the "**sim**" type. That is, the approach of this function is from *'specific to general'*. As said, the model of the null hypothesis is the "**sim**" model whereas the model of the alternative depends on the effect whose omission we want to test.

The collection of Lagrange Multipliers obtained by `Imtestpsur` are standard in the literature and take into account the multivariate nature of the *SUR* model. As a limitation, note that each Multiplier tests for the omission of the same spatial effects in all the cross-sections of the *G* equations.

Usage

```

Imtestpsur(Form = NULL, data = NULL, W = NULL, X = NULL,
  Y = NULL, time = NULL, G = NULL, N = NULL, Tm = NULL,
  print_table = TRUE)

```

Arguments

Form	An object created with the package <code>Formula</code> that describes the model to be estimated. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data	An object of class <code>data.frame</code> or a matrix.
W	A spatial weighting matrix of order ($N \times N$), assumed to be the same for all equations and time periods.
X	A data matrix of order ($NTmGxp$) with the observations of the regressors. The number of covariates in the SUR model is $p = \sum(p_g)$ where p_g is the number of regressors (including the intercept) in the g -th equation, $g = 1, \dots, G$. The specification of <i>X</i> is only necessary if not available a <code>Formula</code> and a data frame. Default = NULL.

Y	A column vector of order $(NTmGxI)$, with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) Cross-sectional/spatial units. The specification of Y is only necessary if not available a Formula and a data frame. Default = NULL.
time	Time variable.
G	Number of equations.
N	Number of cross-section or spatial units
Tm	Number of time periods.
print_table	Logical value to print the output. Default = TRUE

Details

`lmtestspsur` tests for the omission of spatial effects in the "sim" version of the SUR model:

$$y_{tg} = X_{tg}\beta_g + u_{tg}$$

$$E[u_{tg}u'_{th}] = \sigma_{gh}I_N \quad E[u_{tg}u'_{sh}] = 0 \text{ if } t \neq s$$

where y_{tg} and u_{tg} are (NxI) vectors, corresponding to the g-th equation and time period t; X_{tg} is the matrix of exogenous variables, of order (Nxp_g) . Moreover, β_g is an unknown $(p_g \times 1)$ vector of coefficients and $\sigma_{gh}I_N$ the covariance between equations g and h, being σ_{gh} and scalar and I_N the identity matrix of orden N.

The Lagrange Multipliers reported by this function are the followings:

- **LM-SUR-LAG:** Tests for the omission of a spatial lag of the explained variable in the right hand side of the "sim" equation. The model of the alternative is:

$$y_{tg} = \lambda_g W y_{tg} + X_{tg}\beta_g + u_{tg}$$

The null and alternative hypotheses are:

$$H_0 : \lambda_g = 0(\text{forall } g) \text{ vs } H_A : \lambda_g \neq 0(\text{exist } g)$$

- **LM-SUR-ERR:** Tests for the omission of spatial dependence in the equation of the errors of the "sim" model. The model of the alternative is:

$$y_{tg} = X_{tg}\beta_g + u_{tg}; u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

The null and alternative hypotheses are:

$$H_0 : \rho_g = 0(\text{forall } g) \text{ vs } H_A : \rho_g \neq 0(\text{exist } g)$$

- **LM-SUR-SARAR:** Tests for the simultaneous omission of a spatial lag of the explained variable in the right hand side of the "sim" equation and spatial dependence in the equation of the errors. The model of the alternative is:

$$y_{tg} = \lambda_g W y_{tg} + X_{tg}\beta_g + u_{tg}; u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

The null and alternative hypotheses are:

$$H_0 : \lambda_g = \rho_g = 0(\text{forall } g) \text{ vs } H_A : \lambda_g \neq 0 \text{ or } \rho_g \neq 0(\text{exist } g)$$

- **LM*-SUR-SLM and LM*-SUR-SEM:** These two test are the robustified version of the original, raw Multipliers, **LM-SUR-SLM** and **LM-SUR-SEM**, which can be severely oversized if the respective alternative hypothesis is misspecified (this would be the case if, for example, we are testing for omitted lags of the explained variable whereas the problem is that there is spatial dependence in the errors, or viceversa). The null and alternative hypotheses of both test are totally analogous to their twin non robust Multipliers.

Value

A list including:

stat_names	Name of Lagrange Multiplier.
stat	Value of the corresponding Lagrange Multiplier.
df	Degrees of freedom for each Multiplier.

Author(s)

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References

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.
- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.

See Also

[spsurml](#), [lrttestspsur](#)

Examples

```
#####
##### CROSS SECTION DATA (G>1; Tm=1) # #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data("spc")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
LMs <- lmttestspsur(Form = Tformula, data = spc, W = Wspc)

## Not run:
#####
##### PANEL DATA (G>1; Tm>1) #####
#####

#### Example 2: Homicides & Socio-Economics (1960-90)
# Homicides and selected socio-economic characteristics for
```



```

# continental U.S. counties.
# Data for four decennial census years: 1960, 1970, 1980 and 1990.
# https://geodacenter.github.io/data-and-lab/ncovr/

data("NCOVR")
# With different number of exogenous variables in each equation
Tformula <- HR70 | HR80 | HR90 ~ PS70 + UE70 | PS80 + UE80 +RD80 |
          PS90 + UE90 + RD90 + P090
LMs <- lmtestspsur(Form = Tformula, data = NCOVR, W = W)

## End(Not run)

#####
##### PANEL DATA: TEMPORAL CORRELATIONS (G=1; Tm>1) #####
#####

#### Example 3: NCOVR in panel data form
## Takes less than 1 minute
data("NCOVR")
Year <- as.numeric(kronecker(c(1960,1970,1980,1990),matrix(1,nrow = dim(NCOVR)[1])))
HR <- c(NCOVR$HR60,NCOVR$HR70,NCOVR$HR80,NCOVR$HR90)
PS <- c(NCOVR$PS60,NCOVR$PS70,NCOVR$PS80,NCOVR$PS90)
UE <- c(NCOVR$UE60,NCOVR$UE70,NCOVR$UE80,NCOVR$UE90)
NCOVRpanel <- as.data.frame(cbind(Year,HR,PS,UE))
Tformula <- HR ~ PS + UE
LM_time <- lmtestspsur(Form = Tformula, data = NCOVRpanel, time = Year, W = W)

```

lrtestspsur

Likelihood Ratio tests for the specification of spatial SUR models.

Description

The function computes a set of Likelihood Ratio tests, LR, that help the user to select the spatial structure of the SUR model. To achieve this goal, `lrtestspsur` needs to estimate the SUR models "`sim`", "`slm`", "`sem`", "`sdm`", and "`sarar`", using the function `spsurml`.

The five models listed above are related by a nesting sequence, so they can be compared using the adequate LR tests. The function shows the log-likelihood corresponding to the maximum-likelihood estimates and the sequence of LR tests.

Usage

```

lrtestspsur(Form = NULL, data = NULL, W = NULL, X = NULL,
            Y = NULL, time = NULL, G = NULL, N = NULL, Tm = NULL)

```

Arguments

Form	An object created with the package Formula that describes the model to be estimated. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data	An object of class <code>data.frame</code> or a matrix.
W	A spatial weighting matrix of order (NxN) , assumed to be the same for all equations and time periods.
X	A data matrix of order $(NTmGxp)$ with the observations of the regressors. The number of covariates in the SUR model is $p = \text{sum}(p_g)$ where p_g is the number of regressors (including the intercept) in the g -th equation, $g = 1, \dots, G$. The specification of X is only necessary if not available a Formula and a data frame. Default = NULL.
Y	A column vector of order $(NTmGx1)$, with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) Cross-sectional/spatial units. The specification of Y is only necessary if not available a Formula and a data frame. Default = NULL.
time	Time variable.
G	Number of equations.
N	Number of cross-section or spatial units
Tm	Number of time periods.

Details

A fundamental result in maximum-likelihood estimation shows that if *model A* is nested in *model B*, by a set of n restrictions on the parameters of *model B*, then, as the sample size increases, the test statistic: $-2\log[l(H_0)/l(H_A)]$ is a $\chi^2(n)$, being $l(H_0)$ the estimated likelihood under the null hypothesis (*model A*) and $l(H_A)$ the estimated likelihood under the alternative hypothesis (*model B*).

The list of (spatial) models that can be estimated with the function [spsurml](#) includes the following (in addition to the **"slx"** and **"sdem"**):

- **"sim"**: SUR model with no spatial effects

$$y_{tg} = X_{tg}\beta_g + \epsilon_{tg}$$

- **"slm"**: SUR model with spatial lags of the explained variables

$$y_{tg} = \lambda_g W y_{tg} + X_{tg}\beta_g + \epsilon_{tg}$$

- **"sem"**: SUR model with spatial errors

$$y_{tg} = X_{tg}\beta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

- **"sdm"**: SUR model of the Spatial Durbin type

$$y_{tg} = \lambda_g W y_{tg} + X_{tg}\beta_g + W X_{tg}\theta_g + \epsilon_{tg}$$

- **"sarar"**: SUR model with spatial lags of the explained variables and spatial errors

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

This collection of models can be compared, on objective bases, using the LR principle and the following nesting relations:

- **"sim"** vs **"sem"**, where the null hypotheses, in the **"sem"** equation, are:

$$H_0 : \rho_g = 0 \text{ for all } g \text{ vs } H_A : \rho_g \neq 0 \text{ exist } g$$

- **"sim"** vs **"slm"**, where the null hypotheses, in the **"slm"** equation, are:

$$H_0 : \lambda_g = 0 \text{ for all } g \text{ vs } H_A : \lambda_g \neq 0 \text{ exist } g$$

- **"sim"** vs **"sarar"**, where the null hypotheses, in the **"sarar"** equation, are:

$$H_0 : \rho_g = \lambda_g = 0 \text{ for all } g \text{ vs } H_A : \rho_g \neq 0 \text{ or } \lambda_g \neq 0 \text{ exist } g$$

- **"sem"** vs **"sarar"**, where the null hypotheses, in the **"sarar"** equation, are:

$$H_0 : \lambda_g = 0 \text{ for all } g \text{ vs } H_A : \lambda_g \neq 0 \text{ exist } g$$

- **"slm"** vs **"sarar"**, where the null hypotheses, in the **"sarar"** equation, are:

$$H_0 : \rho_g = 0 \text{ for all } g \text{ vs } H_A : \rho_g \neq 0 \text{ exist } g$$

- **"sem"** vs **"sdm"**, also known as *LR-COMFAC*, where the null hypotheses, in the **"sdm"** equation, are:

$$H_0 : -\lambda_g \beta_g = \theta_g \text{ for all } g \text{ vs } H_A : -\lambda_g \beta_g \neq \theta_g \text{ exist } g$$

The degrees of freedom of the corresponding χ^2 distribution is G in the cases of **"sim"** vs **"sem"**, **"sim"** vs **"slm"**, **"sem"** vs **"sarar"**, **"slm"** vs **"sarar"** and **"sem"** vs **"sdm"** and $2G$ in the case of **"sim"** vs **"sarar"**. Moreover, function `lrtestpsur` also returns the p-values associated to the corresponding LR.

Value

`lrtestpsur`, first, prints the value of the estimated log-likelihood for the major spatial specifications. Then, the function shows the values of the LR statistics corresponding to the nested and nesting models compared, together with their associated p-value.

Author(s)

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References

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.
- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.

See Also

[spsurml](#), [lmttestspsur](#)

Examples

```
#####
##### CROSS SECTION DATA (nG=1; nT>1) #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data("spc")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
## It usually requires 1-2 minutes maximum
## LRs <- lrtestspsur(Form = Tformula, data = spc, W = Wspc)

#####
##### CROSS SECTION DATA (nG>1; nT=1) #####
#####

#### Example 2: Homicides & Socio-Economics (1960-90)
# Homicides and selected socio-economic characteristics for
# continental U.S. counties.
# Data for four decennial census years: 1960, 1970, 1980 and 1990.
# https://geodacenter.github.io/data-and-lab/ncovr/

## It could require some minutes
rm(list = ls()) # Clean memory
data("NCOVR")
Tformula <- HR70 | HR80 | HR90 ~ PS70 + UE70 | PS80 + UE80 + RD80 |
          PS90 + UE90 + RD90 + P090
LRs <- lrtestspsur(Form = Tformula, data = NCOVR, W = W)

#####
##### PANEL DATA: TEMPORAL CORRELATIONS (nG=1; nT>1) #####
#####

#### Example 3: Classic panel data

## It could require some minutes
```

```

rm(list = ls()) # Clean memory
data(NCOVR)
N <- nrow(NCOVR)
Tm <- 4
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
pHR <- c(NCOVR$HR60, NCOVR$HR70, NCOVR$HR80, NCOVR$HR90)
pPS <- c(NCOVR$PS60, NCOVR$PS70, NCOVR$PS80, NCOVR$PS90)
pUE <- c(NCOVR$UE60, NCOVR$UE70, NCOVR$UE80, NCOVR$UE90)
pNCOVR <- data.frame(indiv = index_indiv, time = index_time, HR = pHR, PS = pPS, UE = pUE)
rm(NCOVR,pHR,pPS,pUE,index_time,index_indiv)
form_pHR <- HR ~ PS + UE
LRs <- lrtestspsur(Form = form_pHR, data = pNCOVR, W = W, time = pNCOVR$time)

```

lr_betas_spsur	<i>Likelihood ratio for testing homogeneity constraints on beta coefficients of the SUR equations.</i>
----------------	--

Description

Function `lr_betas_spsur` obtains a Likelihood Ratio test, LR in what follows, with the purpose of testing if some of the β coefficients in the G equations of the *SUR* model are equal. This function has a straightforward application, especially when $G = 1$, to the case of testing for the existence of structural breaks in the β parameters.

The function can test for the homogeneity of only one coefficient, of a few of them or even the homogeneity of all the slope terms. The testing procedure implies, *first*, the estimation of both a constrained and a unconstrained model and, *second*, the comparison of the log-likelihoods to compute the LR statistics.

Usage

```

lr_betas_spsur(Form = NULL, data = NULL, R = NULL, b = NULL,
  W = NULL, time = NULL, X = NULL, Y = NULL, G = NULL,
  N = NULL, Tm = NULL, p = NULL, type = "sim",
  printmodels = FALSE, cov = FALSE, trace = FALSE)

```

Arguments

Form	An object created with the package <code>Formula</code> that describes the model to be estimated. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data	An object of class <code>data.frame</code> or a matrix.
R	A row vector of order (<i>l_{xpr}</i>) with the set of r linear constraints on the <i>beta</i> parameters. The <i>first</i> restriction appears in the first p terms, the second restriction in the next p terms and so on. Default = <code>NULL</code> .

b	A column vector of order $(rx1)$ with the values of the linear restrictions on the <i>beta</i> parameters. Default = NULL.
w	A spatial weighting matrix of order (NxN) , assumed to be the same for all equations and time periods.
time	Time variable.
X	A data matrix of order $(NTmGxp)$ with the observations of the regressors. The number of covariates in the SUR model is $p = \text{sum}(p_g)$ where p_g is the number of regressors (including the intercept) in the g -th equation, $g = 1, \dots, G$. The specification of X is only necessary if not available a Formula and a data frame. Default = NULL.
Y	A column vector of order $(NTmGx1)$, with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) Cross-sectional/spatial units. The specification of Y is only necessary if not available a Formula and a data frame. Default = NULL.
G	Number of equations.
N	Number of cross-section or spatial units
Tm	Number of time periods.
p	Number of regressors by equation, including the intercept. p can be a row vector of order $(1xG)$, if the number of regressors is not the same for all the equations, or a scalar, if the G equations have the same number of regressors. The specification of p is only necessary if not available a Formula and a data frame.
type	Type of spatial model specification: " sim ", " slx ", " slm ", " sem ", " sdm ", " sdem " or " sarar ". Default = "sim".
printmodels	Logical value: prints constrained and unconstrained models. Default = FALSE.
cov	Logical value to show the covariance matrix of the <i>beta</i> coefficients. Default = TRUE.
trace	Logical value to show intermediate results during estimation. Default = FALSE.

Value

The function estimates two variants of the same SUR model, namely, a restricted and an unrestricted version. The purpose, as indicated above, is testing homogeneity constraints between the β parameters of the different equations of the SUR. The output of [lr_betas_spsur](#) shows, in first place, the iteration sequence of the maximum-likelihood algorithm. Then appears the Likelihood Ratio, LR, test. The output also includes the maximum-likelihood estimation of the two models.

statistic	The Values of the LR test.
p_val	The p-value of the LR test.
df	Degrees of freedom.
llik_unr	The Log-likelihood of the unrestricted model.
llik_res	The Log-likelihood of the restricted model.

Author(s)

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References

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.

See Also

[spsurml](#), [spsurtime](#), [wald_betas](#)

Examples

```
#####
##### CROSS SECTION DATA (G>1; Tm=1) #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
## H0: equal beta for SMSA in both equations.
R <- matrix(c(0,0,0,1,0,0,0,-1),nrow=1)
b <- matrix(0,ncol=1)
LR_SMSA <- lr_betas_spsur(Form = Tformula, data = spc, W = Wspc,
                          type = "sim", R = R, b = b, trace = TRUE,
                          printmodels = TRUE)

#####
##### PANEL DATA (G>1; Tm>1) #####
#####

#### Example 2: Homicides + Socio-Economics (1960-90)
# Homicides and selected socio-economic characteristics for continental
# U.S. counties.
# Data for four decennial census years: 1960, 1970, 1980 and 1990.
# \url{https://geodacenter.github.io/data-and-lab/ncovr/}

## It usually requires 1-2 minutes maximum
rm(list = ls()) # Clean memory
data(NCOVR)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90
## H0: equal beta for PS80 and PS90 in both equations.
R <- matrix(c(0,1,0,0,-1,0),nrow=1)
```

```

b <- matrix(0,ncol=1)
LR_PS <- lr_betas_spsur(Form = Tformula, data = NCOVR, W = W,
                        type = 'slm', R = R, b = b, printmodels = FALSE)

#####
##### PANEL DATA: TEMPORAL CORRELATIONS (nG=1; nT>1) #####
#####
## Example 3: with classical panel data set. Database is
##           a spatio-temporal panel

### Only execute if you have enough memory...
rm(list = ls()) # Clean memory
data(NCOVR)
N <- nrow(NCOVR)
Tm <- 4
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
pHR <- c(NCOVR$HR60, NCOVR$HR70, NCOVR$HR80, NCOVR$HR90)
pPS <- c(NCOVR$PS60, NCOVR$PS70, NCOVR$PS80, NCOVR$PS90)
pUE <- c(NCOVR$UE60, NCOVR$UE70, NCOVR$UE80, NCOVR$UE90)
pNCOVR <- data.frame(indiv = index_indiv, time = index_time,
                    HR = pHR, PS = pPS, UE = pUE)
rm(NCOVR,pHR,pPS,pUE,index_time,index_indiv)
form_pHR <- HR ~ PS + UE
# H0: equal PS beta coefficient in equations 1, 3, and 4
R <- matrix(0,nrow=2,ncol=12) # nrow = number of restrictions ; ncol = number of beta parameters
R[1,2] <- 1; R[1,8] <- -1 # PS beta coefficient in equations 1 equal to 3
R[2,2] <- 1; R[2,11] <- -1 # PS beta coefficient in equations 1 equal to 4
b <- matrix(0,nrow=2,ncol=1)
lr_partrate <- lr_betas_spsur(Form = form_pHR, data = pNCOVR,
                             time = pNCOVR$time, W = W,
                             type = "slm", R = R, b = b, trace = TRUE,
                             printmodels = FALSE)

```

NCOVR

Homicides in U.S. counties

Description

Homicides and selected socio-economic characteristics for continental U.S. counties. Data for four decennial census years: 1960, 1970, 1980 and 1990.

Usage

NCOVR

Format

A data frame with 3085 rows and 69 variables:

NAME County coded as a name (factor)

STATE_NAME state fips code (factor)

STATE_FIPS state fips code (factor)

CNTY_FIPS county fips code (character)

FIPS combined state and county fips code (character)

STFIPS State fips code (numeric)

COFIPS county fips code (numeric)

FIPSNO fips code as numeric variable

SOUTH dummy variable for Southern counties (South = 1)

HR60, HR70, HR80, HR90 homicide rate per 100,000 (1960, 1970, 1980, 1990)

HC60, HC70, HC80, HC90 homicide count, three year average centered on 1960, 1970, 1980, 1990

PO60, PO70, PO80, PO90 county population, 1960, 1970, 1980, 1990

RD60, RD70, RD80, RD90 resource deprivation 1960, 1970, 1980, 1990 (principal component, see Codebook for details)

PS60, PS70, PS80, PS90 population structure 1960, 1970, 1980, 1990 (principal component, see Codebook for details)

UE60, UE70, UE80, UE90 unemployment rate 1960, 1970, 1980, 1990

DV60, DV70, DV80, DV90 divorce rate 1960, 1970, 1980, 1990 (% males over 14 divorced)

MA60, MA70, MA80, MA90 median age 1960, 1970, 1980, 1990

POL60, POL70, POL80, POL90 log of population 1960, 1970, 1980, 1990

DNL60, DNL70, DNL80, DNL90 log of population density 1960, 1970, 1980, 1990

MFIL59, MFIL69, MFIL79, MFIL89 log of median family income 1960, 1970, 1980, 1990

FP59, FP69, FP79, FP89 % families below poverty 1960, 1970, 1980, 1990 (see Codebook for details)

BLK60, BLK70, BLK80, BLK90 % black 1960, 1970, 1980, 1990

GI59, GI69, GI79, GI89 Gini index of family income inequality 1960, 1970, 1980, 1990

FH60, FH70, FH80, FH90 % female headed households 1960, 1970, 1980, 1990

Source

S. Messner, L. Anselin, D. Hawkins, G. Deane, S. Tolnay, R. Baller (2000). An Atlas of the Spatial Patterning of County-Level Homicide, 1960-1990. Pittsburgh, PA, National Consortium on Violence Research (NCOVR) <https://geodacenter.github.io/data-and-lab/ncovr/>

References

- Baller, R., L. Anselin, S. Messner, G. Deane and D. Hawkins (2001). Structural covariates of US county homicide rates: incorporating spatial effects. *Criminology* 39, 561-590.

```
print.summary.spsur Print method for objects of class summary.spsur.
```

Description

Print method for objects of class `summary.spsur`.

Usage

```
## S3 method for class 'summary.spsur'
print(x, digits = max(3L, getOption("digits") -
  3L), ...)
```

Arguments

<code>x</code>	object of class <code>summary.spsur</code> .
<code>digits</code>	number of digits to show in printed tables. Default: <code>max(3L, getOption("digits") - 3L)</code> .
<code>...</code>	further arguments passed to or from other methods.

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See Also

[summary.spsur](#).

Examples

```
# See examples for \code{\link{spsurm1}} or
# \code{\link{spsur3sls}} functions.
```

spc

A classical Spatial Phillips-Curve

Description

A data set from Anselin (1988, p. 203-2011) used to estimate a Spatial Phillips-Curve for 25 counties in South-West Ohio for two time periods (1981 and 1983).

Usage

```
spc
```

Format

A data frame with 25 rows and 10 variables:

COUNTY County coded as a name.

WAGE83 Changes in wage rates for 1983.

UN83 Inverse unemployment rate in 1983.

NMR83 Net migration rate 1983.

SMSA Dummy variable to identify counties defined as Standard Metropolitan Statistical Areas (SMSA = 1).

WAGE82 Changes in wage rates for 1982.

WAGE81 Changes in wage rates for 1981.

UN80 Inverse unemployment rate in 1980.

NMR80 Net migration rate 1983.

WAGE80 changes in wage rates.

Source

Anselin (1988, p. 203-211)

References

- Anselin, L. (1988). *Spatial Econometrics: Methods and Models*. Springer Science & Business Media.

spsur3sls

Three Stages Least Squares estimation, 3sls, of spatial SUR models.

Description

The function estimates spatial SUR models using three stages least squares, where the instruments are obtained from the spatial lags of the X variables, assumed to be exogenous. The number of equations, time periods and spatial units is not restricted. The user can choose between a Spatial Durbin Model or a Spatial Lag Model, as described below. The estimation procedure allows for the introduction of linear restrictions on the β parameters associated to the regressors.

Usage

```
spsur3sls(Form = NULL, data = NULL, R = NULL, b = NULL, W = NULL,
  X = NULL, Y = NULL, G = NULL, N = NULL, Tm = NULL, p = NULL,
  demean = FALSE, type = "slm", maxlagW = 2)
```

Arguments

Form	An object created with the package Formula that describes the model to be estimated. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data	An object of class <code>data.frame</code> or a matrix.
R	A row vector of order $(I \times r)$ with the set of r linear constraints on the <i>beta</i> parameters. The <i>first</i> restriction appears in the first p terms, the second restriction in the next p terms and so on. Default = NULL.
b	A column vector of order $(r \times 1)$ with the values of the linear restrictions on the <i>beta</i> parameters. Default = NULL.
W	A spatial weighting matrix of order $(N \times N)$, assumed to be the same for all equations and time periods.
X	A data matrix of order $(NTm \times G \times p)$ with the observations of the regressors. The number of covariates in the SUR model is $p = \text{sum}(p_g)$ where p_g is the number of regressors (including the intercept) in the g -th equation, $g = 1, \dots, G$. The specification of X is only necessary if not available a Formula and a data frame. Default = NULL.
Y	A column vector of order $(NTm \times G \times 1)$, with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) Cross-sectional/spatial units. The specification of Y is only necessary if not available a Formula and a data frame. Default = NULL.
G	Number of equations.
N	Number of cross-section or spatial units
Tm	Number of time periods.
p	Number of regressors by equation, including the intercept. p can be a row vector of order $(I \times G)$, if the number of regressors is not the same for all the equations, or a scalar, if the G equations have the same number of regressors. The specification of p is only necessary if not available a Formula and a data frame.
demean	Logical value to allow for the demeaning of panel data. In this case, <code>spsurml</code> subtracts the individual mean to each spatial or cross-sectional unit. Default = FALSE.
type	Type of spatial model, restricted to cases where lags of the explained variable appear in the right hand side of the equations. There are two possibilities: " slm " or " sdm ". Default = "slm".
maxlagW	Maximum spatial lag order of the regressors employed to produce spatial instruments for the spatial lags of the explained variables. Default = 2. Note that in case of $type = "sdm"$, the default value for <code>maxlagW</code> is set to 3 because the first lag of the regressors, WX_{tg} , can not be used as spatial instruments.

Details

`spsur3sls` can be used to estimate two groups of spatial models:

- "**slm**": SUR model with spatial lags of the endogenous in the right hand side of the equations

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + \epsilon_{tg}$$

- **"sdm"**: SUR model of the Spatial Durbin type

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + W X_{tg} \theta_g + \epsilon_{tg}$$

where y_{tg} and ϵ_{tg} are $(Nx1)$ vectors, corresponding to the g -th equation and time period t ; X_{tg} is the matrix of regressors, of order (Nxp_g) . Moreover, λ_g is a spatial coefficient and W is a (NxN) spatial weighting matrix.

By default, the input of this function is an object created with [Formula](#) and a data frame. However, *spsur3s1s* also allows for the direct specification of vector Y and matrix X , with the explained variables and regressors respectively, as inputs (these terms may be the result, for example, of [dgp_spsur](#)).

spsur3s1s is a Least-Squares procedure in three-stages designed to circumvent the endogeneity problems due to the presence of spatial lags of the explained variable in the right hand side of the equations do the SUR. The instruments are produced internally by *spsur3s1s* using a sequence of spatial lags of the X variables, which are assumed to be exogenous. The user must define the number of (spatial) instruments to be used in the procedure, through the argument *maxlagW* (i.e. *maxlagW=3*). Then, the collection of instruments generated is $[W X_{tg}; W * W X_{tg}; W * W * W X_{tg}]$. In the case of a *SDM*, the first lag of the X matrix already is in the equation and cannot be used as instrument. In the example above, the list of instruments for a *SDM* model would be $[W^2 X_{tg}; W^3 X_{tg}]$.

The *first* stage of the procedure consists in the least squares of the Y variables on the set of instruments. From this estimation, the procedure retains the estimates of Y in the so-called *Yls* variables. In the *second* stage, the Y variables that appear in the right hand side of the equation are substituted by *Yls* and the SUR model is estimated by Least Squares. The *third* stage improves the estimates of the second stage through a Feasible Generalized Least Squares estimation of the parameters of the model, using the residuals of the *second* stage to estimate the *Sigma* matrix.

The arguments *R* and *b* allows to introduce linear restrictions on the *beta* coefficients of the G equations. *spsur3s1s*, first, introduces the linear restrictions in the SUR model and builds, internally, the corresponding constrained SUR model. Then, the function estimates the restricted model which is shown in the output. The function does not compute the unconstrained model nor test for the linear restrictions. The user may ask for the unconstrained estimation using another [spsurml](#) estimation. Moreover, the function [wald_betas](#) obtains the Wald test of a set of linear restrictions for an object created previously by [spsurml](#) or [spsur3s1s](#).

Value

Output of the three-stages Least-Squares estimation of the specified spatial model. A list with:

call	Matched call.
type	Type of model specified.
betas	Estimated coefficients for the regressors.
deltas	Estimated spatial coefficients.
se_betas	Estimated standard errors for the estimates of β coefficients.
se_deltas	Estimated standard errors for the estimates of the spatial coefficients.
cov	Estimated covariance matrix for the estimates of <i>beta</i> 's and spatial coefficients.
R2	Coefficient of determination for each equation, obtained as the squared of the correlation coefficient between
Sigma	Estimated covariance matrix for the residuals of the G equations.
Sigma_corr	Estimated correlation matrix for the residuals of the G equations.

Sigma_inv	Inverse of Sigma, the $(G \times G)$ covariance matrix of the residuals of the SUR model.
residuals	Residuals of the model.
df.residuals	Degrees of freedom for the residuals.
fitted.values	Estimated values for the dependent variables.
G	Number of equations.
N	Number of cross-sections or spatial units.
Tm	Number of time periods.
p	Number of regressors by equation (including intercepts).
demean	Logical value used for demeaning.
Y	Vector Y of the explained variables of the SUR model.
X	Matrix X of the regressors of the SUR model.
W	Spatial weighting matrix.

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References

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.
- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.

See Also

[spsurml](#), [wald_betas](#)

Examples

```
#####
##### CROSS SECTION DATA (G=1; Tm>1) #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
## A SUR model without spatial effects
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA

## A SUR-SLM model (3SLS Estimation)
```

```

spcsur.slm.3sls <-spsur3sls(Form = Tformula, data = spc,
                           type = "slm", W = Wspc)
summary(spcsur.slm.3sls)

## A SUR-SDM model (3SLS Estimation)
spcsur.sdm.3sls <-spsur3sls(Form = Tformula, data = spc,
                           type = "sdm", W = Wspc)
summary(spcsur.sdm.3sls)

#####
##### PANEL DATA (G>1; Tm>1) #####
#####

#### Example 2: Homicides + Socio-Economics (1960-90)
# Homicides and selected socio-economic characteristics for continental
# U.S. counties.
# Data for four decennial census years: 1960, 1970, 1980 and 1990.
# https://geodacenter.github.io/data-and-lab/ncovr/
rm(list = ls()) # Clean memory
data(NCOVR)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90

## A SUR-SLR model
NCOVRSUR.slm.3sls <-spsur3sls(Form = Tformula, data = NCOVR, type = "slm",
                              W = W, maxlagW = 2)
summary(NCOVRSUR.slm.3sls)

```

spsurml

Maximum likelihood estimation of spatial SUR model.

Description

This function estimates spatial SUR models using maximum-likelihood methods. The number of equations, time periods and cross-sectional units is not restricted. The user can choose between different spatial specifications as described below. The estimation procedure allows for the introduction of linear restrictions on the β parameters associated to the regressors.

Usage

```

spsurml(Form = NULL, data = NULL, R = NULL, b = NULL, W = NULL,
        X = NULL, Y = NULL, G = NULL, N = NULL, Tm = NULL, p = NULL,
        demean = FALSE, type = "sim", cov = TRUE, control = list(tol =
        0.05, maxit = 200, trace = TRUE))

```

Arguments

Form An object created with the package `Formula` that describes the model to be estimated. This model may contain several responses (explained variables) and a varying number of regressors in each equation.

data	An object of class data.frame or a matrix.
R	A row vector of order $(I \times p_r)$ with the set of r linear constraints on the <i>beta</i> parameters. The <i>first</i> restriction appears in the first p terms, the second restriction in the next p terms and so on. Default = NULL.
b	A column vector of order $(r \times 1)$ with the values of the linear restrictions on the <i>beta</i> parameters. Default = NULL.
W	A spatial weighting matrix of order $(N \times N)$, assumed to be the same for all equations and time periods.
X	A data matrix of order $(NTm \times G \times p)$ with the observations of the regressors. The number of covariates in the SUR model is $p = \text{sum}(p_g)$ where p_g is the number of regressors (including the intercept) in the g -th equation, $g = 1, \dots, G$. The specification of X is only necessary if not available a Formula and a data frame. Default = NULL.
Y	A column vector of order $(NTm \times G \times 1)$, with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) Cross-sectional/spatial units. The specification of Y is only necessary if not available a Formula and a data frame. Default = NULL.
G	Number of equations.
N	Number of cross-section or spatial units
Tm	Number of time periods.
p	Number of regressors by equation, including the intercept. p can be a row vector of order $(I \times G)$, if the number of regressors is not the same for all the equations, or a scalar, if the G equations have the same number of regressors. The specification of p is only necessary if not available a Formula and a data frame.
demean	Logical value to allow for the demeaning of panel data. In this case, <code>spsurml</code> subtracts the individual mean to each spatial or cross-sectional unit. Default = FALSE.
type	Type of spatial model specification: " sim ", " slx ", " slm ", " sem ", " sdm ", " sdem " or " sarar ". Default = " sim ".
cov	Logical value to show the covariance matrix of the <i>beta</i> coefficients. Default = TRUE.
control	List of additional control arguments.

Details

The list of (spatial) models that can be estimated with the `spsurml` function are:

- "**sim**": SUR model with no spatial effects

$$y_{tg} = X_{tg}\beta_g + \epsilon_{tg}$$

- "**slx**": SUR model with spatial lags of the regressors

$$y_{tg} = X_{tg}\beta_g + W X_{tg}\theta_g + \epsilon_{tg}$$

- **"slm"**: SUR model with spatial lags of the explained variables

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + \epsilon_{tg}$$

- **"sem"**: SUR model with spatial errors

$$y_{tg} = X_{tg} \beta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

- **"sdm"**: SUR model of the Spatial Durbin type

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + W X_{tg} \theta_g + \epsilon_{tg}$$

- **"sdem"**: SUR model with spatial lags of the regressors and spatial errors

$$y_{tg} = X_{tg} \beta_g + W X_{tg} \theta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

- **"sarar"**: SUR model with spatial lags of the explained variables and spatial errors

$$y_{tg} = \lambda_g W y_{tg} + X_{tg} \beta_g + u_{tg}$$

$$u_{tg} = \rho_g W u_{tg} + \epsilon_{tg}$$

Value

Output of the maximum-likelihood estimation of the specified spatial SUR model. A list with:

call	Matched call.
type	Type of model specified.
betas	Estimated coefficients for the regressors.
deltas	Estimated spatial coefficients.
se_betas	Estimated standard errors for the estimates of <i>beta</i> .
se_deltas	Estimated standard errors for the estimates of the spatial coefficients.
cov	Estimated covariance matrix for the estimates of <i>beta</i> 's and spatial coefficients.
llsur	Value of the likelihood function at the maximum-likelihood estimates.
R2	Coefficient of determination for each equation, obtained as the squared of the correlation coefficient between
Sigma	Estimated covariance matrix for the residuals of the <i>G</i> equations.
Sigma_corr	Estimated correlation matrix for the residuals of the <i>G</i> equations.
Sigma_inv	Inverse of Sigma, the (<i>GxG</i>) covariance matrix of the residuals of the SUR model.
residuals	Residuals of the model.
df.residuals	Degrees of freedom for the residuals.
fitted.values	Estimated values for the dependent variables.
BP	Value of the Breusch-Pagan statistic to test the null hypothesis of diagonality among the errors of the <i>G</i> equations.
LMM	Marginal Lagrange Multipliers, $LM(\rho \lambda)$ and $LM(\lambda \rho)$, to test for omitted spatial effects in the specification.
G	Number of equations.
N	Number of cross-sections or spatial units.
Tm	Number of time periods.
p	Number of regressors by equation (including intercepts).
demean	Logical value used for demeaning.
Y	Vector <i>Y</i> of the explained variables of the SUR model.
X	Matrix <i>X</i> of the regressors of the SUR model.

Control arguments

- tol Numerical value for the tolerance for the estimation algorithm until convergence. Default = 1e-3.
maxit Maximum number of iterations until convergence; it must be an integer value. Default = 200.
trace A logical value to show intermediate results during the estimation process. Default = *TRUE*.

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References

- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.
- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.
- Breusch T, Pagan A (1980) The Lagrange multiplier test and its applications to model specification in econometrics. *Rev Econ Stud* 47: 239-254

See Also

[spsur3sls](#), [lmtestspsur](#), [wald_betas](#), [lrtestspsur](#)

Examples

```
#####
##### CROSS SECTION DATA (G>1; Tm=1) #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
## It usually requires 2-3 minutes maximum...
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
spcsur.sim <-spsurml(Form = Tformula, data = spc, type = "sim", W = Wspc)
summary(spcsur.sim)

## A SUR-SLX model
```

```

spsur.slx <-spsurml(Form = Tformula, data = spc, type = "slx", W = Wspc)
summary(spsur.slx)

## A SUR-SLM model
spsur.slm <-spsurml(Form = Tformula, data = spc, type = "slm", W = Wspc)
summary(spsur.slm)
rm(spsur.slm) # remove

## A SUR-SEM model
spsur.sem <-spsurml(Form = Tformula, data = spc, type = "sem", W = Wspc)
summary(spsur.sem)
rm(spsur.sem) # remove

## A SUR-SDM model
spsur.sdm <-spsurml(Form = Tformula, data = spc, type = "sdm", W = Wspc)
summary(spsur.sdm)
rm(spsur.sdm) # remove

## A SUR-SDM model
spsur.sdem <-spsurml(Form = Tformula, data = spc, type = "sdem", W = Wspc)
summary(spsur.sdem)
rm(spsur.sdem) # remove

## A SUR-SARAR model
spsur.sarar <-spsurml(Form = Tformula, data = spc, type = "sarar", W = Wspc)
summary(spsur.sarar)
rm(spsur.sarar) # remove

#####
##### G=1; Tm>1 #####
#####

#### Example 2: Homicides + Socio-Economics (1960-90)
# Homicides and selected socio-economic characteristics for continental
# U.S. counties.
# Data for four decennial census years: 1960, 1970, 1980 and 1990.
# \url{https://geodacenter.github.io/data-and-lab/ncovr/}

## It usually requires 1-2 minutes maximum...
rm(list = ls()) # Clean memory
data(NCOVR)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90
## A SUR-SIM model
NCOVRSUR.sim <-spsurml(Form = Tformula, data = NCOVR, type = "sim", W = W)
summary(NCOVRSUR.sim)
rm(NCOVRSUR.sim)

## A SUR-SLX model
NCOVRSUR.slx <-spsurml(Form = Tformula, data = NCOVR, type = "slx", W = W)
summary(NCOVRSUR.slx)
rm(NCOVRSUR.slx)

```

```

## It usually requires 1-2 minutes maximum...
## A SUR-SLM model
NCOVRSUR.slm <-spsurml(Form = Tformula, data = NCOVR, type = "slm", W = W)
summary(NCOVRSUR.slm)
rm(NCOVRSUR.slm)

## A SUR-SDM model
NCOVRSUR.sdm <-spsurml(Form = Tformula, data = NCOVR, type = "sdm", W = W)
summary(NCOVRSUR.sdm)
rm(NCOVRSUR.sdm)

## A SUR-SEM model
NCOVRSUR.sem <-spsurml(Form = Tformula, data = NCOVR, type = "sem", W = W)
summary(NCOVRSUR.sem)
## A SUR-SDEM model
NCOVRSUR.sdem <-spsurml(Form = Tformula, data = NCOVR, type = "sdem",W = W)
summary(NCOVRSUR.sdem)

## A SUR-SARAR model
NCOVRSUR.sarar <-spsurml(Form = Tformula, data = NCOVR,
                          type = "sarar", W = W)
summary(NCOVRSUR.sarar)

#####
##### SUR with G>1; Tm>1 ###
#####

## It usually requires 2-3 minutes maximum...
rm(list = ls()) # Clean memory
#### Reshape NCOVR in panel format
data(NCOVR,package="spsur")
N <- nrow(NCOVR)
Tm <- 4
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
pHR <- c(NCOVR$HR60, NCOVR$HR70, NCOVR$HR80, NCOVR$HR90)
pPS <- c(NCOVR$PS60, NCOVR$PS70, NCOVR$PS80, NCOVR$PS90)
pUE <- c(NCOVR$UE60, NCOVR$UE70, NCOVR$UE80, NCOVR$UE90)
pDV <- c(NCOVR $DV60, NCOVR$DV70, NCOVR$DV80, NCOVR$DV90)
pFP <- c(NCOVR$FP59, NCOVR$FP70, NCOVR$FP80, NCOVR$FP90)
pSOUTH <- rep(NCOVR$SOUTH, Tm)
pNCOVR <- data.frame(indiv = index_indiv, time = index_time,
                    HR = pHR, PS = pPS, UE = pUE, DV = pDV,
                    FP = pFP, SOUTH = pSOUTH)
rm(NCOVR) # Free memory...
pform <- HR | DV | FP ~ PS + UE | PS + UE + SOUTH | PS

## SIM (easy to compute...)

psur_sim <- spsurml(Form = pform, data = pNCOVR, W = W, type = "sim")
summary(psur_sim)

```

```

## SLM (cov = FALSE to prevent overflows of memory)
psur_slm <- spsurml(Form = pform, data = pNCOVR, W = W,
                    type = "slm", cov = FALSE)

psur_slm$deltas
psur_slm$betas
psur_slm$Sigma_corr
rm(psur_slm)
## SEM (cov = FALSE to prevent overflows of memory)
### Only execute if you have enough memory...
psur_sem <- spsurml(Form = pform, data = pNCOVR, W = W,
                    type = "sem", cov = FALSE)

psur_sem$deltas
psur_sem$betas
psur_sem$Sigma_corr
psur_sem

#####
##### Demeaning Examples with G>1; Tm>>1 #####
#####

#' rm(list = ls()) # Clean memory
set.seed(123456)
Tm <- 10 # Number of time periods
G <- 3 # Number of equations
N <- 100 # Number of spatial elements
p <- 3 # Number of independent variables
Sigma <- matrix(0.5, ncol = G, nrow = G)
diag(Sigma) <- 1
Betas <- rep(1:3, G)
lambda <- 0.5
rho <- 0.0 # spatial autocorrelation error term = 0
# random coordinates
co <- cbind(runif(N,0,1),runif(N,0,1))
W <- spdep::nb2mat(spdep::knn2nb(spdep::knearneigh(co, k = 5, longlat = FALSE)))
DGPsim <- dgp_spsur(Sigma = Sigma, Betas = Betas, rho = rho, lambda = lambda,
                  Tm = Tm, G = G, N = N, p = p, W = W)

## SLM without demeaning

SUR_slm <-spsurml(Y= DGPsim$Y, X = DGPsim$X, G = G, N = N, Tm = Tm,
                 p = p, W = W, type = "slm")
summary(SUR_slm)

# SLM with demeaning

SUR_slm_dem <-spsurml(Y= DGPsim$Y, X = DGPsim$X, G = G, N = N, Tm = Tm,
                    p = p, W = W, type = "slm", demean = TRUE)
summary(SUR_slm_dem)

```

spsurtime

Estimation of SUR models for simple spatial panels ($G=1$).

Description

This function estimates SUR models for simple spatial panel datasets. `spsurtime` is restricted, specifically, to cases where there is only one equation, $G=1$, and a varying number of spatial units, N , and time periods, Tm . The SUR structure appears in form of serial dependence among the error terms corresponding to the same spatial unit. Note that it is assumed that all spatial units share a common pattern of serial dependence.

The user can choose between different types of spatial specifications, as described below, and the estimation algorithms allow for the introduction of linear restrictions on the β parameters associated to the regressors. The spatial panels with SUR structure can be estimated by maximum-likelihood methods or three-stages least squares procedures, using spatial instrumental variables.

Usage

```
spsurtime(Form, data, time, type = "sim", method = "ml", maxlagW = 2,
           W = NULL, cov = TRUE, demean = FALSE, trace = TRUE, R = NULL,
           b = NULL)
```

Arguments

Form	An object created with the package <code>Formula</code> that describes the model to be estimated. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data	An object of class <code>data.frame</code> or a matrix.
time	Time variable.
type	Type of spatial model specification: " <code>sim</code> ", " <code>slx</code> ", " <code>slm</code> ", " <code>sem</code> ", " <code>sdm</code> ", " <code>sdem</code> " or " <code>sarar</code> ". Default = " <code>sim</code> ".
method	Method of estimation for the spatial panel SUR model, either <code>ml</code> or <code>3sls</code> . Default = <code>ml</code> .
maxlagW	Maximum spatial lag order of the regressors employed to produce spatial instruments for the spatial lags of the explained variables. Default = 2. Note that in case of <code>type="sdm"</code> , the default value for <code>maxlagW</code> is set to 3 because the first lag of the regressors, WX_{tg} , can not be used as spatial instruments.
W	A spatial weighting matrix of order ($N \times N$), assumed to be the same for all equations and time periods.
cov	Logical value to show the covariance matrix of the <i>beta</i> coefficients. Default = TRUE.
demean	Logical value to allow for the demeaning of panel data, subtracting the individual mean to each spatial or cross-sectional unit. Default = FALSE.
trace	Logical value to show intermediate results. Default = TRUE.

R	A row vector of order ($l \times p$) with the set of r linear constraints on the <i>beta</i> parameters. The <i>first</i> restriction appears in the first p terms, the second restriction in the next p terms and so on. Default = NULL.
b	A column vector of order ($r \times 1$) with the values of the linear restrictions on the <i>beta</i> parameters. Default = NULL.

Details

Function `spsurtime` only admits a formula, created with `Formula` and a dataset of class `data.frame` or matrix. That is, the data cannot be uploaded using data matrices Y and X provided for other functions in this package.

The argument *time* selects the variable, in the `data.frame`, associated to the time dimension in the panel dataset. Then `spsurtime` operates as in Anselin (1988), that is, each cross-section is treated as if it were an equation in a SUR model, which now has Tm 'equations' and N individuals.

The SUR structure appears because there is serial dependence in the errors of each individual in the panel. The serial dependence in the errors is not parameterized, but estimated non-parametrically in the *Sigma* covariance matrix returned by the function. An important constraint to mention is that the serial dependence assumed to be the same for all individuals in the sample. Serial dependence among individuals is excluded from Anselin approach.

Value

Output of the maximum-likelihood or three-stages least-squares estimation of the spatial panel SUR model. The final list depends of the estimation method but, typically, you will find information about:

call	Matched call.
type	Type of model specified.
betas	Estimated coefficients for the regressors.
deltas	Estimated spatial coefficients.
se_betas	Estimated standard errors for the estimates of <i>beta</i> .
se_deltas	Estimated standard errors for the estimates of the spatial coefficients.
cov	Estimated covariance matrix for the estimates of <i>beta</i> 's and spatial coefficients.
llsur	Value of the likelihood function at maximum-likelihood estimation. Only if <i>method = ml</i> .
R2	Global coefficient of determination for the Tm equations, obtained as the squared of the correlation coefficient.
Sigma	Estimated covariance matrix for the residuals of the G equations.
Sigma_corr	Estimated correlation matrix for the residuals of the G equations.
Sigma_inv	Inverse of Sigma, the $(G \times G)$ covariance matrix of the residuals of the SUR model.
residuals	Residuals of the model.
df.residuals	Degrees of freedom for the residuals.
fitted.values	Estimated values for the dependent variables.
BP	Value of the Breusch-Pagan statistic to test the null hypothesis of diagonality among the errors of the G equations.
LMM	Marginal Lagrange Multipliers, $LM(\rho \lambda)$ and $LM(\lambda \rho)$, to test for omitted spatial effects in the specification.
N	Number of cross-sections or spatial units.
Tm	Number of time periods.
demean	Logical value used for demeaning.
W	Spatial weighting matrix.

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References

- Anselin, L. (1988). *Spatial econometrics: methods and models*. Dordrecht, Kluwer Academic Publishers.
- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.
- López, F.A., Martínez-Ortiz, P.J., & Cegarra-Navarro, J.G. (2017). Spatial spillovers in public expenditure on a municipal level in Spain. *Annals of Regional Science*, 58(1), 39-65.
- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.

See Also

[spsurml](#), [spsur3sls](#), [wald_betas](#), [lmtestpsur](#), [lrtestpsur](#)

Examples

```
#####
##### PANEL DATA (G=1; Tm>1) #####
#####

## Example 1:
rm(list = ls()) # Clean memory
N <- nrow(spc)
Tm <- 2
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
WAGE <- c(spc$WAGE83, spc$WAGE81)
UN <- c(spc$UN83, spc$UN80)
NMR <- c(spc$NMR83, spc$NMR80)
SMSA <- c(spc$SMSA, spc$SMSA)
pspc <- data.frame(index_indiv, index_time, WAGE, UN, NMR, SMSA)
form_pspc <- WAGE ~ UN + NMR + SMSA

# SLM by 3SLS
pspc_slm <- spsurtime(Form = form_pspc, data = pspc, W = Wspc,
                      time = pspc$index_time, type = "slm", method = "3sls")
summary(pspc_slm)
```



```

## Example 2:
rm(list = ls()) # Clean memory
data(NCOVR,package="spsur")
N <- nrow(NCOVR)
Tm <- 4
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
pHR <- c(NCOVR$HR60, NCOVR$HR70, NCOVR$HR80, NCOVR$HR90)
pPS <- c(NCOVR$PS60, NCOVR$PS70, NCOVR$PS80, NCOVR$PS90)
pUE <- c(NCOVR$UE60, NCOVR$UE70, NCOVR$UE80, NCOVR$UE90)
pNCOVR <- data.frame(indiv = index_indiv, time = index_time,
                    HR = pHR, PS = pPS, UE = pUE)

rm(NCOVR)
form_pHR <- HR ~ PS + UE

# SIM by ML

## pHR_sim <- spsurtime(Form = form_pHR, data = pNCOVR, W = W,
##                       time = pNCOVR$time, type = "sim", method = "ml")
## summary(pHR_sim)

# SLM by 3SLS

pHR_slm <- spsurtime(Form = form_pHR, data = pNCOVR, W = W,
                    time = pNCOVR$time, type = "slm", method = "3sls")
summary(pHR_slm)

##### LR tests about betas in spatio-temporal models

## Usually takes less than 1 minute
## H0: equal PS and UE beta in equations 3 and 4 al
R <- matrix(0,nrow=2,ncol=12)
R[1,8] <- 1; R[1,11] <- -1
R[2,9] <- 1; R[2,12] <- -1
b <- matrix(0, nrow = 2, ncol = 1)
lr_partrate <- lr_betas_spsur(Form = form_pHR, data = pNCOVR,
                             time = pNCOVR$time, W = W,
                             type = "sim", R = R, b = b, trace = TRUE,
                             printmodels = TRUE)

##### Wald tests about betas in spatio-temporal models
wald_betas(pHR_sim , R = R , b = b) # SIM model
wald_betas(pHR_slm , R = R , b = b) # SLM model
##### Wald tests about spatial-parameters in
##### spatio-temporal models
## H0: equal lambdas in slm model for equations 1 and 2.
R2 <- matrix(0, nrow = 1, ncol = 4)
R2[1,1] <- 1; R2[1,2] <- -1
b2 <- matrix(0, nrow = 1, ncol = 1)
wald_deltas(pHR_slm, R = R2, b = b2)

```

`summary.spsur`*Summary of estimated objects of class spsur.*

Description

This function summarizes estimated *spsur* objects. The tables in the output include basic information for each equation. The report also shows other complementary results corresponding to the SUR model like the $(G \times G)$ covariance matrix of the residuals of the equations of the SUR, the estimated log-likelihood, the Breusch-Pagan diagonality test or the Marginal Lagrange Multiplier, LMM, tests of spatial dependence.

Usage

```
## S3 method for class 'spsur'  
summary(object, ...)
```

Arguments

<code>object</code>	<i>spsur</i> object estimated using spsurml , spsur3sls or spsurtime functions.
<code>...</code>	further arguments passed to or from other methods.

Value

An object of class *summary.spsur*

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See Also

[print.summary.spsur](#); [spsurml](#); [spsur3sls](#).

Examples

```
# See examples for \code{\link{spsurml}} or  
# \code{\link{spsur3sls}} functions.
```

W *Spatial weight matrix for U.S. Counties*

Description

A spatial weight matrix row-standardized based on first order contiguity criterium.

Usage

W

Format

A row-standardized squared matrix with 3085 rows and columns. The rows and columns follow the same order than Counties included in [NCOVR](#) data frame.

Source

<https://geodacenter.github.io/data-and-lab/ncovr/>

References

- Baller, R., L. Anselin, S. Messner, G. Deane and D. Hawkins (2001). Structural covariates of US county homicide rates: incorporating spatial effects. *Criminology* 39, 561-590.

wald_betas *Wald tests on the beta coefficients of the equation of the SUR model*

Description

The function [wald_betas](#) can be seen as a complement to the restricted estimation procedures included in the functions [spsurml](#) and [spsur3sls](#). [wald_betas](#) obtains Wald tests for sets of linear restrictions on the coefficients β of the SUR model. The restrictions may involve coefficients of the same equation or coefficients from different equations. The function has great flexibility in this respect. Note that [wald_betas](#) is more general than [lr_betas_spsur](#) in the sense that the last function only allows to test for restrictions of homogeneity of subsets of β coefficients among the different equations in the SUR model, and in a maximum-likelihood framework.

In order to work with [wald_betas](#), the model on which the linear restrictions are to be tested needs to exist as an *spsur* object. Using the information contained in the object, [wald_betas](#) obtains the corresponding Wald statistic for the null hypotheses specified by the user through the *R* row vector and *b* column vector, used also in [spsurml](#) and [spsur3sls](#). The function shows the value of the Wald test statistics and its associated p-values.

Usage

```
wald_betas(results, R, b)
```

Arguments

- `results` : An object created with `spsurml` or `spsur3sls`. This argument serves the user to indicate the spatial SUR model, previously estimated by maximum-likelihood or 3sls, where the set of linear restrictions are to be tested.
- `R` : A row vector of order $(1 \times P \times r)$ showing the set of r linear constraints on the β parameters. The *first* restriction appears in the first K terms in R , the *second* restriction in the next K terms and so on. Default = NULL.
- `b` : A column vector of order $(r \times 1)$ with the values of the linear restrictions on the β parameters.

Value

The output of the function is very simple and consists of two pieces of information, the value of the Wald statistic and the corresponding p-value, plus the degrees of freedom of the test.

<code>Wald stat</code>	The value of Wald test.
<code>p_val</code>	The p-value of Wald test.
<code>q</code>	Degrees of freedom of the corresponding χ^2 distribution.

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References

- López, F.A., Mur, J., and Angulo, A. (2014). Spatial model selection strategies in a SUR framework. The case of regional productivity in EU. *Annals of Regional Science*, 53(1), 197-220.
- López, F.A., Martínez-Ortiz, P.J., & Cegarra-Navarro, J.G. (2017). Spatial spillovers in public expenditure on a municipal level in Spain. *Annals of Regional Science*, 58(1), 39-65.
- Mur, J., López, F., and Herrera, M. (2010). Testing for spatial effects in seemingly unrelated regressions. *Spatial Economic Analysis*, 5(4), 399-440.

See Also

`spsurml`, `spsur3sls`, `lr_betas_spsur`

Examples

```
#####
```

```
##### CROSS SECTION DATA (G=1; Tm>1) #####
#####

#### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
## Estimate SUR-SLM model
spcsur.slm <- spsur3sls(Form = Tformula, data = spc, type = "slm", W = Wspc)
summary(spcsur.slm)
## H_0: equality between SMSA coefficients in both equations.
R1 <- matrix(c(0,0,0,1,0,0,0,-1), nrow=1)
b1 <- matrix(0, ncol=1)
Wald_beta <- wald_betas(results = spcsur.slm, R = R1, b = b1)

## H_0: equality between intercepts and SMSA coefficients in both equations.
R2 <- matrix(c(1,0,0,0,-1,0,0,0,0,0,1,0,0,0,-1),
             nrow = 2, ncol = 8, byrow = TRUE)
b2 <- matrix(c(0,0),ncol=1)
wald_betas(results = spcsur.slm, R = R2, b = b2)
#####
##### G=1; Tm>1 #####
#####

#### Example 2: Homicides + Socio-Economics (1960-90)

## Usually takes 1-2 minutes maximum
data(NCOVR)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90
#####
## A SUR-SLM model
NCOVRSUR.slm <- spsurml(Form = Tformula, data = NCOVR, type = "slm", W = W)
summary(NCOVRSUR.slm)
R1 <- matrix(c(0,1,0,0,-1,0), nrow=1)
b1 <- matrix(0, ncol=1)
wald_betas(results = NCOVRSUR.slm, R = R1, b = b1)
```

wald_deltas

Wald tests for spatial parameters coefficients.

Description

Function `wald_deltas` obtains Wald tests for linear restrictions on the spatial coefficients of a SUR model that has been estimated previously through the function `spsurml`. The restrictions can affect to coefficients of the same equation (i.e., $\lambda_g = \rho_g \text{ for all } g$) or can involve coefficients from different equations (i.e., $\lambda_g = \lambda_h$). The function has great flexibility in this respect. Note that `wald_deltas` only works in a maximum-likelihood framework.

In order to work with `wald_betas`, the model on which the linear restrictions are to be tested needs to exist as an `spsur` object. Using the information contained in the object, `wald_deltas` obtains

the corresponding Wald statistic for the null hypotheses specified by the user through the R row vector and b column vector discussed, used also in [spsurml](#). The function shows the resulting Wald test statistics and their corresponding p-values.

Usage

```
wald_deltas(results, R, b)
```

Arguments

- results** : An object created with [spsurml](#) or [spsur3sls](#). This argument serves the user to indicate the spatial SUR model, previously estimated by maximum-likelihood or 3sls, where the set of linear restrictions are to be tested.
- R** : A row vector of order $(IxGr)$ or $(Ix2Gr)$ showing the set of r linear constraints on the spatial parameters. The last case is reserved to "**sarar**" models where there appear G parameters λ_g and G parameters ρ_g , $2G$ spatial parameters in total. The *first* restriction appears in the first G terms in R ($2G$ for the "**sarar**" case), the second restriction in the next G terms ($2G$ for the "**sarar**" case) and so on. Default = NULL.
- b** : A column vector of order $(rx1)$ with the values of the linear restrictions on the β parameters. Default = NULL.

Value

The output of the function is very simple and consists of two pieces of information, the value of the Wald statistic and the corresponding p-value, plus the degrees of freedom of the test.

wald_stat	The value of Wald test.
p_val	The p-value of Wald test.
q	Degrees of freedom of the corresponding χ^2 distribution.

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See Also

[spsurml](#), [spsur3sls](#)

Examples

```
#####
##### CROSS SECTION DATA (G>1; Tm=1) #####
#####
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA

#####
## Estimate SUR-SLM model
spcsur.slm <-spsur3sls(Form = Tformula, data = spc, type = "slm", W= Wspc)
summary(spcsur.slm)
## H_0: equality of the lambda parameters of both equations.
R1 <- matrix(c(1,-1), nrow=1)
b1 <- matrix(0, ncol=1)
wald_deltas(results = spcsur.slm, R = R1, b = b1)

#####
## Estimate SUR-SEM model
#' ## It usually requires 1-2 minutes maximum
spcsur.sem <-spsurml(Form = Tformula, data = spc, type = "sem", W = Wspc)
summary(spcsur.sem)
## H_0: equality of the rho parameters of both equations.
R2 <- matrix(c(1,-1), nrow=1)
b2 <- matrix(0, ncol=1)
wald_deltas(results = spcsur.sem, R = R2, b = b2)

#####
## Estimate SUR-SARAR model
## It usually requires 2-3 minutes maximum
spcsur.sarar <-spsurml(Form = Tformula, data = spc,
                      type = "sarar", W = Wspc)
summary(spcsur.sarar)
## H_0: equality of the lambda and rho parameters of both equations.
R3 <- matrix(c(1,-1,0,0,0,0,1,-1),nrow=2,ncol=4,byrow=TRUE)
b3 <- matrix(c(0,0), ncol=1)
wald_deltas(results = spcsur.sarar, R = R3, b = b3)

#####
##### G=1; Tm>1 #####
#####

#' ##### Example 2: Homicides + Socio-Economics (1960-90)
#' # It could make an error out-of-memory in some computers
rm(list = ls()) # Clean memory
data(NCOVR)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90

#####
## A SUR-SLM model
NCOVRSUR.slm <-spsurml(Form = Tformula, data = NCOVR, type = "slm", W = W)
summary(NCOVRSUR.slm)
## H_0: equality of the lambda parameters of both equations.
R1 <- matrix(c(1,-1), nrow=1)
```

```

b1 <- matrix(0, ncol=1)
wald_deltas(results = NCOVRSUR.slm, R = R1, b = b1)

#####
## Estimate SUR-SEM model
NCOVRSUR.sem <- spsurml(Form = Tformula, data = NCOVR, type = "sem", W = W)
summary(NCOVRSUR.sem)
## H_0: equality of the rho parameters of both equations.
R2 <- matrix(c(1,-1), nrow=1)
b2 <- matrix(0, ncol=1)
wald_deltas(results = NCOVRSUR.sem, R = R2, b = b2)

```

Wspc

Spatial weight matrix for South-West Ohio Counties to estimate Spatial Phillips-Curve

Description

A spatial weight matrix row-standardized based on first order contiguity criterium.

Usage

Wspc

Format

A row-standardized squared matrix with 25 rows and columns. The rows and columns follow the same order than provinces included in *spc* data frame.

Source

Anselin (1988, p. 207)

References

- Anselin, L. (1988). *Spatial Econometrics: Methods and Models*. Springer Science & Business Media.

Index

*Topic **datasets**

NCOVR, 24

spc, 26

W, 43

Wspc, 48

dgp_spsur, 5, 6, 8, 29

dunif, 7

Formula, 5, 14, 15, 18, 21, 22, 28, 29, 31, 32,
38, 39

impacts, 4, 10, 12

lmttestspsur, 3, 14, 15, 20, 34, 40

lr_betas_spsur, 21, 21, 22, 43, 44

lrttestspsur, 16, 17, 17, 19, 34, 40

NCOVR, 24, 43

print.summary.spsur, 26, 42

spc, 26

spsur-package, 2

spsur3sls, 4, 9, 13, 27, 29, 34, 40, 42–44, 46

spsurml, 4, 9, 13, 14, 16–18, 20, 23, 28–30,
31, 32, 40, 42–46

spsurtime, 9, 23, 38, 38, 39, 42

summary.spsur, 26, 42

W, 43

wald_betas, 4, 23, 29, 30, 34, 40, 43, 43, 45

wald_deltas, 4, 45, 45

Wspc, 48