Package 'susieR'

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Type Package

Title Sum of Single Effects Linear Regression

Description Implements methods for variable selection in linear regression based on the ``Sum of Single Effects'' (SuSiE) model, as described in Wang et al (2020) <DOI:10.1101/501114>. These methods provide simple summaries, called ``Credible Sets'', for accurately quantifying uncertainty in which variables should be selected. The methods are motivated by genetic fine-mapping applications, and are particularly well-suited to settings where variables are highly correlated and detectable effects are sparse. The fitting algorithm, a Bayesian analogue of stepwise selection methods called ``Iterative Bayesian Stepwise Selection'' (IBSS), is simple and fast, allowing the SuSiE model be fit to large data sets (thousands of samples and hundreds of thousands of variables).

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R topics documented:

`coef.susie` *Extract regression coefficients from susie fit*

Description

Extract regression coefficients from susie fit

Usage

```
## S3 method for class 'susie'
coef(object, ...)
```

Arguments

object	A susie fit.
...	Additional arguments passed to the generic coef method.

Value

A p+1 vector, the first element being an intercept, and the remaining p elements being estimated regression coefficients.

---

compute_ss *Compute sufficient statistics for input to* susie_suff_stat

---

Description

This is a synonym for compute_suff_stat included for historical reasons (deprecated).

Usage

```
compute_ss(X, y, standardize = FALSE)
```

Arguments

X	An n by p matrix of covariates.
y	An n vector.
standardize	Logical flag indicating whether to standardize columns of X to unit variance prior to computing summary data.

Value

A list of sufficient statistics (X'X, X'y, y'y and n)

Examples

```
data(N2finemapping)
ss = compute_ss(N2finemapping$X, N2finemapping$Y[,1])
```

compute_suff_stat *Compute sufficient statistics for input to* susie_suff_stat

Description

Computes the sufficient statistics $X'X, X'y, y'y$ and n after centering (and possibly standardizing) the columns of X and centering y to have mean zero. We also store the column means of X and mean of y.

Usage

```
compute_suff_stat(X, y, standardize = FALSE)
```

Arguments

X	An n by p matrix of covariates.
y	An n vector.
standardize	Logical flag indicating whether to standardize columns of X to unit variance prior to computing summary data

Value

A list of sufficient statistics (XtX,Xty,yty,n) and X_colmeans, y_mean.

Examples

```
data(N2finemapping)
ss = compute_suff_stat(N2finemapping$X, N2finemapping$Y[,1])
```

estimate_s_rss *Estimate s in* susie_rss *Model Using Regularized LD*

Description

The estimated s gives information about the consistency between the z scores and LD matrix. A larger s means there is a strong inconsistency between z scores and LD matrix. The "null-mle" method obtains mle of s under $z|R\ N(0,(1-s)R+sI), 0<s<1$. The "null-partialmle" method obtains mle of s under $U^Tz|R\ N(0,sI)$, in which U is a matrix containing the of eigenvectors that span the null space of R; that is, the eigenvectors corresponding to zero eigenvalues of R. The estimated s from "null-partialmle" could be greater than 1. The "null-pseudomle" method obtains mle of s under pseudolikelihood $L(s)=\prod_{j=1}^{p}p(z_j|z_{-j},s,R), 0<s<1$.

Usage

```
estimate_s_rss(z, R, r_tol = 1e-08, method = "null-mle")
```

Arguments

z	A p-vector of z scores.
R	A p by p symmetric, positive semidefinite correlation matrix.
r_tol	Tolerance level for eigenvalue check of positive semidefinite matrix of R.
method	a string specifies the method to estimate s.

Value

A number between 0 and 1.

Examples

```
set.seed(1)
n = 500
p = 1000
beta = rep(0,p)
beta[1:4] = 0.01
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
input_ss = compute_suff_stat(X,y,standardize = TRUE)
ss = univariate_regression(X,y)
R = cor(X)
attr(R,"eigen") = eigen(R, symmetric = TRUE)
zhat = with(ss,betahat/sebetahat)
s = estimate_s_rss(zhat, R)
```

FinemappingConvergence

Simulated Fine-mapping Data with Convergence Problem.

Description

Data simulated using real genotypes from 50,000 individuals and 200 SNPs. Two of the SNPs have non-zero effects on the multivariate response. The response data are generated under a linear regression model. The simulated response and the columns of the genotype matrix are centered.

Format

FinemappingConvergence is a list with the following elements:

XtX Summary statistics computed using the centered and scaled genotype matrix.

Xty Summary statistics computed using the centered and scaled genotype data, and the centered simulated response.

yty yty is computed using the centered simulated response.

n The sample size (50,000).

true_coef The coefficients used to simulate the responses.

z z-scores from a simple (single-SNP) linear regression.

Examples

```
data(FinemappingConvergence)
```

`get_cs_correlation` *Get Correlations Between CSs, using Variable with Maximum PIP From Each CS*

Description

This function evaluates the correlation between single effect CSs. It is not part of the SuSiE inference. Rather, it is designed as a diagnostic tool to assess how correlated the reported CS are.

Usage

```
get_cs_correlation(model, X = NULL, Xcorr = NULL, max = FALSE)
```

Arguments

`model`	A SuSiE fit, typically an output from `susie` or one of its variants.
`X`	n by p matrix of values of the p variables (covariates) in n samples. When provided, correlation between variables will be computed and used to remove CSs whose minimum correlation among variables is smaller than `min_abs_corr`.
`Xcorr`	p by p matrix of correlations between variables (covariates). When provided, it will be used to remove CSs whose minimum correlation among variables is smaller than `min_abs_corr`.
`max`	When `max = FAFLSE`, return a matrix of CS correlations. When `max = TRUE`, return only the maximum absolute correlation among all pairs of correlations.

Value

A matrix of correlations between CSs, or the maximum absolute correlation when `max = TRUE`.

kriging_rss *Compute Distribution of z-scores of Variant j Given Other z-scores, and Detect Possible Allele Switch Issue*

Description

Under the null, the rss model with regularized LD matrix is $z|R, s\ N(0, (1-s)R + sI))$. We use a mixture of normals to model the conditional distribution of z_j given other z scores, $z_j|z_{-j}, R, s\ \sum_{k=1}^{K} \pi_k N(-\Omega_{j,-j} z_{-j}/\Omega_{j}$ $\Omega = ((1-s)R + sI)^{-1}$, $\sigma_1, ..., \sigma_k$ is a grid of fixed positive numbers. We estimate the mixture weights π We detect the possible allele switch issue using likelihood ratio for each variant.

Usage

```
kriging_rss(
  z,
  R,
  r_tol = 1e-08,
  s = estimate_s_rss(z, R, r_tol, method = "null-mle")
)
```

Arguments

z	A p-vector of z scores.
R	A p by p symmetric, positive semidefinite correlation matrix.
r_tol	Tolerance level for eigenvalue check of positive semidefinite matrix of R.
s	an estimated s from `estimate_s_rss`

Value

a list containing a ggplot2 plot object and a table. The plot compares observed z score vs the expected value. The possible allele switched variants are labeled as red points (log LR > 2 and abs(z) > 2). The table summarizes the conditional distribution for each variant and the likelihood ratio test.

Examples

```
set.seed(1)
n = 500
p = 1000
beta = rep(0,p)
beta[1:4] = 0.01
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
ss = univariate_regression(X,y)
R = cor(X)
attr(R,"eigen") = eigen(R, symmetric = TRUE)
```

```
zhat = with(ss,betahat/sebetahat)
condz = kriging_rss(zhat, R)
condz$plot
```

`N2finemapping` *Simulated Fine-mapping Data with Two Effect Variables*

Description

This data set contains a genotype matrix for 574 individuals and 1,002 variables. The variables are genotypes after centering and scaling, and therefore retain the correlation structure of the original genotype data. Two of the variables have non-zero effects on the multivariate response. The response data are generated under a multivariate linear regression model. See Wang *et al* (2020) for details.

Format

`N2finemapping` is a list with the following elements:

X Centered and scaled genotype data.

chrom Chromomsome of the original data, in hg38 coordinates.

pos Chromomosomal position of the original data, in hg38 coordinates. The information can be used to compare impact of using other genotype references of the same variables in `susie_rss` application.

true_coef Simulated effect sizes.

residual_variance Simulated residual covariance matrix.

Y Simulated multivariate response.

allele_freq Allele frequencies based on the original genotype data.

V Suggested prior covariance matrix for effect sizes of the two non-zero effect variables.

References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* doi: 10.1101/501114.

Examples

```
data(N2finemapping)
```

N3finemapping *Simulated Fine-mapping Data with Three Effect Variables.*

Description

The data-set contains a matrix of 574 individuals and 1,001 variables. These variables are real-world genotypes centered and scaled, and therefore retains the correlation structure of variables in the original genotype data. 3 out of the variables have non-zero effects. The response data is generated under a multivariate linear regression model. See Wang *et al* (2020) for more details.

Format

`N3finemapping` is a list with the following elements:

X N by P variable matrix of centered and scaled genotype data.

chrom Chromomsome of the original data, in hg38 coordinate.

pos Chromomosomal positoin of the original data, in hg38 coordinate. The information can be used to compare impact of using other genotype references of the same variables in susie_rss application.

true_coef The simulated effect sizes.

residual_variance The simulated residual covariance matrix.

Y The simulated response variables.

allele_freq Allele frequency of the original genotype data.

V Prior covariance matrix for effect size of the three non-zero effect variables.

References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* doi: 10.1101/501114.

Examples

```
data(N3finemapping)
```

predict.susie *Predict outcomes or extract coefficients from susie fit.*

Description

Predict outcomes or extract coefficients from susie fit.

Usage

```
## S3 method for class 'susie'
predict(object, newx = NULL, type = c("response", "coefficients"), ...)
```

Arguments

object	A susie fit.
newx	A new value for X at which to do predictions.
type	The type of output. For type = "response", predicted or fitted outcomes are returned; for type = "coefficients", the estimated coefficients are returned.
...	Other arguments used by generic predict function. These extra arguments are not used here.

Value

For type = "response", predicted or fitted outcomes are returned; for type = "coefficients", the estimated coefficients are returned. If the susie fit has intercept = NA (which is common when using susie_suff_stat) then predictions are computed using an intercept of 0, and a warning is emitted.

summary.susie *Summarize Susie Fit.*

Description

summary method for the "susie" class.

Usage

```
## S3 method for class 'susie'
summary(object, ...)

## S3 method for class 'summary.susie'
print(x, ...)
```

Arguments

object	A susie fit.
...	Additional arguments passed to the generic summary or print.summary method.
x	A susie summary.

Value

summary.susie returns a list containing a data frame of variables and a data frame of credible sets.

SummaryConsistency *Simulated Fine-mapping Data with LD matrix From Reference Panel.*

Description

Data simulated using real genotypes from 10,000 individuals and 200 SNPs. One SNP have non-zero effect on the multivariate response. The response data are generated under a linear regression model. There is also one SNP with flipped allele between summary statistics and the reference panel.

Format

SummaryConsistency is a list with the following elements:

z z-scores computed by fitting univariate simple regression variable-by-variable.

ldref LD matrix estimated from the reference panel.

flip_id The index of the SNP with the flipped allele.

signal_id The index of the SNP with the non-zero effect.

Examples

```
data(SummaryConsistency)
```

susie *Sum of Single Effects (SuSiE) Regression*

Description

Performs a sparse Bayesian multiple linear regression of y on X, using the "Sum of Single Effects" model from Wang et al (2020). In brief, this function fits the regression model $y = \mu + Xb + e$, where elements of e are *i.i.d.* normal with zero mean and variance `residual_variance`, μ is an intercept term and b is a vector of length p representing the effects to be estimated. The "susie assumption" is that $b = \sum_{l=1}^{L} b_l$ where each b_l is a vector of length p with exactly one non-zero element. The prior on the non-zero element is normal with zero mean and variance `var(y) * scaled_prior_variance`. The value of `L` is fixed, and should be chosen to provide a reasonable upper bound on the number of non-zero effects to be detected. Typically, the hyperparameters `residual_variance` and `scaled_prior_variance` will be estimated during model fitting, although they can also be fixed as specified by the user. See functions `susie_get_cs` and other functions of form `susie_get_*` to extract the most commonly-used results from a susie fit.

Usage

```
susie(
  X,
  y,
  L = min(10, ncol(X)),
  scaled_prior_variance = 0.2,
  residual_variance = NULL,
  prior_weights = NULL,
  null_weight = NULL,
  standardize = TRUE,
  intercept = TRUE,
  estimate_residual_variance = TRUE,
  estimate_prior_variance = TRUE,
  estimate_prior_method = c("optim", "EM", "simple"),
  check_null_threshold = 0,
  prior_tol = 1e-09,
  residual_variance_upperbound = Inf,
  s_init = NULL,
  coverage = 0.95,
  min_abs_corr = 0.5,
  compute_univariate_zscore = FALSE,
  na.rm = FALSE,
  max_iter = 100,
  tol = 0.001,
  verbose = FALSE,
  track_fit = FALSE,
  residual_variance_lowerbound = var(drop(y))/10000,
  refine = FALSE
)
```

```
susie_suff_stat(
  bhat,
  shat,
  R,
  n,
  var_y,
  XtX,
  Xty,
  yty,
  X_colmeans = NA,
  y_mean = NA,
  maf = NULL,
  maf_thresh = 0,
  L = 10,
  scaled_prior_variance = 0.2,
  residual_variance = NULL,
  estimate_residual_variance = TRUE,
  estimate_prior_variance = TRUE,
  estimate_prior_method = c("optim", "EM", "simple"),
  check_null_threshold = 0,
  prior_tol = 1e-09,
  r_tol = 1e-08,
  prior_weights = NULL,
  null_weight = NULL,
  standardize = TRUE,
  max_iter = 100,
  s_init = NULL,
  coverage = 0.95,
  min_abs_corr = 0.5,
  tol = 0.001,
  verbose = FALSE,
  track_fit = FALSE,
  check_input = FALSE,
  refine = FALSE
)
```

Arguments

X	An n by p matrix of covariates.
y	The observed responses, a vector of length n.
L	Maximum number of non-zero effects in the susie regression model. If L is larger than the number of covariates, p, L is set to p.
scaled_prior_variance	The prior variance, divided by `var(y)` (or by `(1/(n-1))yty` for `susie_suff_stat`); that is, the prior variance of each non-zero element of b is `var(y) * scaled_prior_variance`. The value provided should be either a scalar or a vector of length `L`. If `estimate_prior_variance = TRUE`, this provides initial estimates of the prior variances.

residual_variance
: Variance of the residual. If estimate_residual_variance = TRUE, this value provides the initial estimate of the residual variance. By default, it is set to var(y) in susie and (1/(n-1))yty in susie_suff_stat.

prior_weights
: A vector of length p, in which each entry gives the prior probability that corresponding column of X has a nonzero effect on the outcome, y.

null_weight
: Prior probability of no effect (a number between 0 and 1, and cannot be exactly 1).

standardize
: If standardize = TRUE, standardize the columns of X to unit variance prior to fitting (or equivalently standardize XtX and Xty to have the same effect). Note that scaled_prior_variance specifies the prior on the coefficients of X *after* standardization (if it is performed). If you do not standardize, you may need to think more carefully about specifying scaled_prior_variance. Whatever your choice, the coefficients returned by coef are given for X on the original input scale. Any column of X that has zero variance is not standardized.

intercept
: If intercept = TRUE, the intercept is fitted; it intercept = FALSE, the intercept is set to zero. Setting intercept = FALSE is generally not recommended.

estimate_residual_variance
: If estimate_residual_variance = TRUE, the residual variance is estimated, using residual_variance as an initial value. If estimate_residual_variance = FALSE, the residual variance is fixed to the value supplied by residual_variance.

estimate_prior_variance
: If estimate_prior_variance = TRUE, the prior variance is estimated (this is a separate parameter for each of the L effects). If provided, scaled_prior_variance is then used as an initial value for the optimization. When estimate_prior_variance = FALSE, the prior variance for each of the L effects is determined by the value supplied to scaled_prior_variance.

estimate_prior_method
: The method used for estimating prior variance. When estimate_prior_method = "simple" is used, the likelihood at the specified prior variance is compared to the likelihood at a variance of zero, and the setting with the larger likelihood is retained.

check_null_threshold
: When the prior variance is estimated, compare the estimate with the null, and set the prior variance to zero unless the log-likelihood using the estimate is larger by this threshold amount. For example, if you set check_null_threshold = 0.1, this will "nudge" the estimate towards zero when the difference in log-likelihoods is small. A note of caution that setting this to a value greater than zero may lead the IBSS fitting procedure to occasionally decrease the ELBO.

prior_tol
: When the prior variance is estimated, compare the estimated value to prior_tol at the end of the computation, and exclude a single effect from PIP computation if the estimated prior variance is smaller than this tolerance value.

residual_variance_upperbound
: Upper limit on the estimated residual variance. It is only relevant when estimate_residual_variance = TRUE.

s_init
: A previous susie fit with which to initialize.

coverage — A number between 0 and 1 specifying the "coverage" of the estimated confidence sets.

min_abs_corr — Minimum absolute correlation allowed in a credible set. The default, 0.5, corresponds to a squared correlation of 0.25, which is a commonly used threshold for genotype data in genetic studies.

compute_univariate_zscore — If compute_univariate_zscore = TRUE, the univariate regression z-scores are outputted for each variable.

na.rm — Drop any missing values in y from both X and y.

max_iter — Maximum number of IBSS iterations to perform.

tol — A small, non-negative number specifying the convergence tolerance for the IBSS fitting procedure. The fitting procedure will halt when the difference in the variational lower bound, or "ELBO" (the objective function to be maximized), is less than tol.

verbose — If verbose = TRUE, the algorithm's progress, and a summary of the optimization settings, are printed to the console.

track_fit — If track_fit = TRUE, trace is also returned containing detailed information about the estimates at each iteration of the IBSS fitting procedure.

residual_variance_lowerbound — Lower limit on the estimated residual variance. It is only relevant when estimate_residual_variance = TRUE.

refine — If refine = TRUE, then an additional iterative refinement procedure is used, after the IBSS algorithm, to check and escape from local optima (see details).

bhat — A p-vector of estimated effects.

shat — A p-vector of standard errors.

R — A p by p symmetric, positive semidefinite matrix. It can be $X'X$, the covariance matrix $X'X/(n-1)$, or a correlation matrix. It should be estimated from the same samples used to compute bhat and shat. Using an out-of-sample matrix may produce unreliable results.

n — The sample size.

var_y — The sample variance of y, defined as $y'y/(n-1)$. When the sample variance cannot be provided, the coefficients (returned from coef) are computed on the "standardized" X, y scale.

XtX — A p by p matrix $X'X$ in which the columns of X are centered to have mean zero.

Xty — A p-vector $X'y$ in which y and the columns of X are centered to have mean zero.

yty — A scalar $y'y$ in which y is centered to have mean zero.

X_colmeans — A p-vector of column means of X. If both X_colmeans and y_mean are provided, the intercept is estimated; otherwise, the intercept is NA.

y_mean — A scalar containing the mean of y. If both X_colmeans and y_mean are provided, the intercept is estimated; otherwise, the intercept is NA.

maf — Minor allele frequency; to be used along with maf_thresh to filter input summary statistics.

maf_thresh	Variants having a minor allele frequency smaller than this threshold are not used.
r_tol	Tolerance level for eigenvalue check of positive semidefinite matrix of R.
check_input	If check_input = TRUE, susie_suff_stat performs additional checks on XtX and Xty. The checks are: (1) check that XtX is positive semidefinite; (2) check that Xty is in the space spanned by the non-zero eigenvectors of XtX.

Details

The function susie implements the IBSS algorithm from Wang et al (2020). The option refine = TRUE implements an additional step to help reduce problems caused by convergence of the IBSS algorithm to poor local optima (which is rare in our experience, but can provide misleading results when it occurs). The refinement step incurs additional computational expense that increases with the number of CSs found in the initial run.

The function susie_suff_stat implements essentially the same algorithms, but using sufficient statistics. (The statistics are sufficient for the regression coefficients b, but not for the intercept μ; see below for how the intercept is treated.) If the sufficient statistics are computed correctly then the results from susie_suff_stat should be the same as (or very similar to) susie, although runtimes will differ as discussed below. The simplest sufficient statistics are the sample size n, and then the p by p matrix $X'X$, the p-vector $X'y$, and the sum of squared y values $y'y$, all computed after centering the columns of X and the vector y to have mean 0; these can be computed using compute_suff_stat. Alternatively the user can provide n and bhat (the univariate OLS estimates from regressing y on each column of X), shat (the standard errrors from these OLS regressions), the p by p symmetric, positive semidefinite correlation (or covariance) matrix $R = (1/(n-1))X'X$, and the variance of y, again all computed from centered X and y. Note that here R and bhat should be computed using the same matrix X. If you do not have access to the original X to compute the matrix R then use susie_rss.

The handling of the intercept term in susie_suff_stat needs some additional explanation. Computing the summary data after centering X and y effectively ensures that the resulting posterior quantities for b allow for an intercept in the model; however, the actual value of the intercept cannot be estimated from these centered data. To estimate the intercept term the user must also provide the column means of X and the mean of y (X_colmeans and y_mean). If these are not provided, they are treated as NA, which results in the intercept being NA. If for some reason you prefer to have the intercept be 0 instead of NA then set X_colmeans = 0, y_mean = 0.

For completeness, we note that if susie_suff_stat is run on $X'X, X'y, y'y$ computed *without* centering X and y, and with X_colmeans = 0, y_mean = 0, this is equivalent to susie applied to X, y with intercept = FALSE (although results may differ due to different initializations of residual_variance and scaled_prior_variance). However, this usage is not recommended for for most situations.

The computational complexity of susie is $O(npL)$ per iteration, whereas susie_suff_stat is $O(p^2L)$ per iteration (not including the cost of computing the sufficient statistics, which is dominated by the $O(np^2)$ cost of computing $X'X$). Because of the cost of computing $X'X$, susie will usually be faster. However, if $n >> p$, and/or if $X'X$ is already computed, then susie_suff_stat may be faster.

Value

A "susie" object with some or all of the following elements:

alpha	An L by p matrix of posterior inclusion probabilites.
mu	An L by p matrix of posterior means, conditional on inclusion.
mu2	An L by p matrix of posterior second moments, conditional on inclusion.
Xr	A vector of length n, equal to `X %*% colSums(alpha * mu)`.
lbf	log-Bayes Factor for each single effect.
lbf_variable	log-Bayes Factor for each variable and single effect.
intercept	Intercept (fixed or estimated).
sigma2	Residual variance (fixed or estimated).
V	Prior variance of the non-zero elements of b, equal to `scaled_prior_variance * var(y)`.
elbo	The value of the variational lower bound, or "ELBO" (objective function to be maximized), achieved at each iteration of the IBSS fitting procedure.
fitted	Vector of length n containing the fitted values of the outcome.
sets	Credible sets estimated from model fit; see `susie_get_cs` for details.
pip	A vector of length p giving the (marginal) posterior inclusion probabilities for all p covariates.
z	A vector of univariate z-scores.
niter	Number of IBSS iterations that were performed.
converged	`TRUE` or `FALSE` indicating whether the IBSS converged to a solution within the chosen tolerance level.

`susie_suff_stat` returns also outputs:

XtXr	A p-vector of `t(X)` times the fitted values, `X %*% colSums(alpha*mu)`.

References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* **82**, 1273-1300 doi: 10.1101/501114.

See Also

`susie_get_cs` and other `susie_get_*` functions for extracting results; `susie_trendfilter` for applying the SuSiE model to non-parametric regression, particularly changepoint problems, and `susie_rss` for applying the SuSiE model when one only has access to limited summary statistics related to X and y (typically in genetic applications).

Examples

```
# susie example
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
```

```
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res1 = susie(X,y,L = 10)
susie_get_cs(res1) # extract credible sets from fit
plot(beta,coef(res1)[-1])
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
plot(y,predict(res1))
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")

# susie_suff_stat example
input_ss = compute_suff_stat(X,y)
res2 = with(input_ss,
            susie_suff_stat(XtX = XtX,Xty = Xty,yty = yty,n = n,
                            X_colmeans = X_colmeans,y_mean = y_mean,L = 10))
plot(coef(res1),coef(res2))
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
```

`susie_auto` *Attempt at Automating SuSiE for Hard Problems*

Description

`susie_auto` is an attempt to automate reliable running of susie even on hard problems. It implements a three-stage strategy for each L: first, fit susie with very small residual error; next, estimate residual error; finally, estimate the prior variance. If the last step estimates some prior variances to be zero, stop. Otherwise, double L, and repeat. Initial runs are performed with relaxed tolerance; the final run is performed using the default susie tolerance.

Usage

```
susie_auto(
  X,
  y,
  L_init = 1,
  L_max = 512,
  verbose = FALSE,
  init_tol = 1,
  standardize = TRUE,
  intercept = TRUE,
  max_iter = 100,
  tol = 0.01,
  ...
)
```

Arguments

X	An n by p matrix of covariates.
y	The observed responses, a vector of length n.
L_init	The initial value of L.
L_max	The largest value of L to consider.
verbose	If verbose = TRUE, the algorithm's progress, and a summary of the optimization settings, are printed to the console.
init_tol	The tolerance to passed to susie during early runs (set large to shorten the initial runs).
standardize	If standardize = TRUE, standardize the columns of X to unit variance prior to fitting. Note that scaled_prior_variance specifies the prior on the coefficients of X *after* standardization (if it is performed). If you do not standardize, you may need to think more carefully about specifying scaled_prior_variance. Whatever your choice, the coefficients returned by coef are given for X on the original input scale. Any column of X that has zero variance is not standardized.
intercept	If intercept = TRUE, the intercept is fitted; it intercept = FALSE, the intercept is set to zero. Setting intercept = FALSE is generally not recommended.
max_iter	Maximum number of IBSS iterations to perform.
tol	A small, non-negative number specifying the convergence tolerance for the IBSS fitting procedure. The fitting procedure will halt when the difference in the variational lower bound, or "ELBO" (the objective function to be maximized), is less than tol.
...	Additional arguments passed to susie.

Value

See susie for a description of return values.

See Also

susie

Examples

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = susie_auto(X,y)
plot(beta,coef(res)[-1])
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
plot(y,predict(res))
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
```

susie_get_objective *Inferences From Fitted SuSiE Model*

Description

These functions access basic properties or draw inferences from a fitted susie model.

Usage

```
susie_get_objective(res, last_only = TRUE, warning_tol = 1e-06)

susie_get_posterior_mean(res, prior_tol = 1e-09)

susie_get_posterior_sd(res, prior_tol = 1e-09)

susie_get_niter(res)

susie_get_prior_variance(res)

susie_get_residual_variance(res)

susie_get_lfsr(res)

susie_get_posterior_samples(susie_fit, num_samples)

susie_get_cs(
  res,
  X = NULL,
  Xcorr = NULL,
  coverage = 0.95,
  min_abs_corr = 0.5,
  dedup = TRUE,
  squared = FALSE
)

susie_get_pip(res, prune_by_cs = FALSE, prior_tol = 1e-09)
```

Arguments

res	A susie fit, typically an output from susie or one of its variants. For susie_get_pip and susie_get_cs, this may instead be the posterior inclusion probability matrix, alpha.
last_only	If last_only = FALSE, return the ELBO from all iterations; otherwise return the ELBO from the last iteration only.
warning_tol	Warn if ELBO is decreasing by this tolerance level.
prior_tol	Filter out effects having estimated prior variance smaller than this threshold.

susie_fit	A susie fit, an output from susie.
num_samples	The number of draws from the posterior distribution.
X	n by p matrix of values of the p variables (covariates) in n samples. When provided, correlation between variables will be computed and used to remove CSs whose minimum correlation among variables is smaller than min_abs_corr.
Xcorr	p by p matrix of correlations between variables (covariates). When provided, it will be used to remove CSs whose minimum correlation among variables is smaller than min_abs_corr.
coverage	A number between 0 and 1 specifying desired coverage of each CS.
min_abs_corr	A "purity" threshold for the CS. Any CS that contains a pair of variables with correlation less than this threshold will be filtered out and not reported.
dedup	If dedup = TRUE, remove duplicate CSs.
squared	If squared = TRUE, report min, mean and median of squared correlation instead of the absolute correlation.
prune_by_cs	Whether or not to ignore single effects not in a reported CS when calculating PIP.

Value

susie_get_objective returns the evidence lower bound (ELBO) achieved by the fitted susie model and, optionally, at each iteration of the IBSS fitting procedure.

susie_get_residual_variance returns the (estimated or fixed) residual variance parameter.

susie_get_prior_variance returns the (estimated or fixed) prior variance parameters.

susie_get_posterior_mean returns the posterior mean for the regression coefficients of the fitted susie model.

susie_get_posterior_sd returns the posterior standard deviation for coefficients of the fitted susie model.

susie_get_niter returns the number of model fitting iterations performed.

susie_get_pip returns a vector containing the posterior inclusion probabilities (PIPs) for all variables.

susie_get_lfsr returns a vector containing the average lfsr across variables for each single-effect, weighted by the posterior inclusion probability (alpha).

susie_get_posterior_samples returns a list containing the effect sizes samples and causal status with two components: b, an num_variables x num_samples matrix of effect sizes; gamma, an num_variables x num_samples matrix of causal status random draws.

susie_get_cs returns credible sets (CSs) from a susie fit, as well as summaries of correlation among the variables included in each CS. If desired, one can filter out CSs that do not meet a specified "purity" threshold; to do this, either X or Xcorr must be supplied. It returns a list with the following elements:

cs	A list in which each list element is a vector containing the indices of the variables in the CS.
coverage	The nominal coverage specified for each CS.

purity	If X or Xcorr iis provided), the purity of each CS.
cs_index	If X or Xcorr is provided) the index (number between 1 and L) of each reported CS in the supplied susie fit.

Examples

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
s = susie(X,y,L = 10)
susie_get_objective(s)
susie_get_objective(s, last_only=FALSE)
susie_get_residual_variance(s)
susie_get_prior_variance(s)
susie_get_posterior_mean(s)
susie_get_posterior_sd(s)
susie_get_niter(s)
susie_get_pip(s)
susie_get_lfsr(s)
```

---

susie_init_coef *Initialize a susie object using regression coefficients*

Description

Initialize a susie object using regression coefficients

Usage

```
susie_init_coef(coef_index, coef_value, p)
```

Arguments

coef_index	An L-vector containing the the indices of the nonzero coefficients.
coef_value	An L-vector containing initial coefficient estimates.
p	A scalar giving the number of variables.

Value

A list with elements alpha, mu and mu2 to be used by susie.

Examples

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[sample(1:1000,4)] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))

# Initialize susie to ground-truth coefficients.
s = susie_init_coef(which(beta != 0),beta[beta != 0],length(beta))
res = susie(X,y,L = 10,s_init=s)
```

---

susie_plot *SuSiE Plots.*

Description

`susie_plot` produces a per-variable summary of the SuSiE credible sets. `susie_plot_iteration` produces a diagnostic plot for the susie model fitting. For `susie_plot_iteration`, several plots will be created if `track_fit = TRUE` when calling `susie`.

Usage

```
susie_plot(
  model,
  y,
  add_bar = FALSE,
  pos = NULL,
  b = NULL,
  max_cs = 400,
  add_legend = NULL,
  ...
)

susie_plot_iteration(model, L, file_prefix, pos = NULL)
```

Arguments

`model`	A SuSiE fit, typically an output from `susie` or one of its variants. For `suse_plot`, the susie fit must have `model$z`, `model$PIP`, and may include `model$sets`. `model` may also be a vector of z-scores or PIPs.
`y`	A string indicating what to plot: either `"z"` for z-scores, `"PIP"` for posterior inclusion probabilities, `"log10PIP"` for posterior inclusion probabiliities on the (base-10) log-scale. For any other setting, the data are plotted as is.

add_bar	If add_bar = TRUE, add horizontal bar to signals in credible interval.
pos	Indices of variables to plot. If pos = NULL all variables are plotted.
b	For simulated data, set b = TRUE to highlight "true" effects (highlights in red).
max_cs	The largest credible set to display, either based on purity (set max_cs between 0 and 1), or based on size (set max_cs > 1).
add_legend	If add_legend = TRUE, add a legend to annotate the size and purity of each CS discovered. It can also be specified as location where legends should be added, e.g., add_legend = "bottomright" (default location is "topright").
...	Additional arguments passed to plot.
L	An integer specifying the number of credible sets to plot.
file_prefix	Prefix to path of output plot file. If not specified, the plot, or plots, will be saved to a temporary directory generated using tempdir.

Value

Invisibly returns NULL.

See Also

susie_plot_changepoint

Examples

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[sample(1:1000,4)] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = susie(X,y,L = 10)
susie_plot(res,"PIP")
susie_plot(res,"PIP",add_bar = TRUE)
susie_plot(res,"PIP",add_legend = TRUE)
susie_plot(res,"PIP", pos=1:500, add_legend = TRUE)
# Plot selected regions with adjusted x-axis position label
res$genomic_position = 1000 + (1:length(res$pip))
susie_plot(res,"PIP",add_legend = TRUE,
           pos = list(attr = "genomic_position",start = 1000,end = 1500))
# True effects are shown in red.
susie_plot(res,"PIP",b = beta,add_legend = TRUE)

set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[sample(1:1000,4)] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
```

```
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = susie(X,y,L = 10)
susie_plot_iteration(res, L=10)
```

`susie_plot_changepoint` *Plot changepoint data and susie fit using ggplot2*

Description

Plots original data, y, overlaid with line showing susie fitted value and shaded rectangles showing credible sets for changepoint locations.

Usage

```
susie_plot_changepoint(
  s,
  y,
  line_col = "blue",
  line_size = 1.5,
  cs_col = "red"
)
```

Arguments

`s`	A susie fit generated by `susie_trendfilter(y,order = 0)`.
`y`	An n-vector of observations that are ordered in time or space (assumed equally-spaced).
`line_col`	Color for the line showing fitted values.
`line_size`	Size of the lines showing fitted values
`cs_col`	Color of the shaded rectangles showing credible sets.

Value

A ggplot2 plot object.

Examples

```
set.seed(1)
mu = c(rep(0,50),rep(1,50),rep(3,50),rep(-2,50),rep(0,300))
y = mu + rnorm(500)
# Here we use a less sensitive tolerance so that the example takes
# less time; in practice you will likely want to use a more stringent
# setting such as tol = 0.001.
s = susie_trendfilter(y,tol = 0.1)
```

```
# Produces ggplot with credible sets for changepoints.
susie_plot_changepoint(s,y)
```

`susie_rss` *Sum of Single Effects (SuSiE) Regression using summary statistics*

Description

`susie_rss` performs variable selection under a sparse Bayesian multiple linear regression of Y on X using only the z-scores from standard univariate regression of Y on each column of X, and an estimate R of the correlation matrix between columns of X. It does this by combining the "RSS likelihood" from Zhu and Stephens (2017) with the Sum of Single Effects" model from Wang et al (2020).

Usage

```
susie_rss(
  z,
  R,
  z_ld_weight = 0,
  L = 10,
  prior_variance = 50,
  estimate_prior_variance = TRUE,
  ...
)
```

Arguments

`z`	A p-vector of z scores.
`R`	A p by p symmetric, positive semidefinite correlation matrix.
`z_ld_weight`	This parameter is included for backwards compatibility with previous versions of the function, but it is no longer recommended to use a non-zero value. If `z_ld_weight > 0`, the matrix R used in the model is adjusted to be `cov2cor((1-w)*R + w*tcrossprod(z))`, where `w = z_ld_weight`.
`L`	Maximum number of components (nonzero coefficients) in the susie regression model. If L is larger than the number of covariates, p, L is set to p.
`prior_variance`	The prior variance(s) for the non-zero element of b_l. It is either a scalar, or a vector of length L. When `estimate_prior_variance = TRUE` (highly recommended) this simply provides an initial value for the prior variance, and the default value of 50 is simply intended to be a large initial value.
`estimate_prior_variance`	If `estimate_prior_variance = TRUE`, which is highly recommended, the prior variance is estimated (this is a separate parameter for each of the L effects). If provided, `prior_variance` is then used as an initial value for the optimization. When `estimate_prior_variance = FALSE` (not recommended) the prior variance for each of the L effects is determined by the value supplied to `prior_variance`.

... Other parameters to be passed to susie_suff_stat.

Details

In some applications, particularly genetic applications, it is desired to fit a regression model ($Y = X\tilde{b} + E$ say, which we refer to as "the original regression model" or ORM) without access to the actual values of Y and X, but given only some summary statistics. susie_rss assumes the availability of z scores from standard univariate regression of Y on each column of X, and an estimate R of the correlation matrix between columns of X (R is sometimes called the LD matrix in genetic applications). See Zhu and Stephens (2017), and references therein, for further background.

The susie_rss function is based on the model (2.10) from Zhu and Stephens, $z|R, b\ N(Rb, R)$ where b is a vector of length p representing the effects to be estimated. The effect b_j is simply a multiple of the coefficient $\tilde{b}_j$ in the ORM, and so b_j is non-zero if and only if $\tilde{b}_j$ is non-zero. In this sense the variable selection problem in this model is the same as the variable selection problem in the ORM, and so the credible sets and PIPs computed by susie_rss can be interpreted as credible sets and PIPs for the ORM. However, converting posterior estimates of b_j to estimates of $\tilde{b}_j$ would require computation of the scaling factor (not done here).

More precisely, susie_rss assumes the log-likelihood for b is $l(b; z, R) = -0.5(b'Rb - 2z'b)$, which is equivalent to model (2.10) from Zhu and Stephens if R is invertible, but does not require R to be invertible. It combines this likelihood with the "susie prior" which assumes that $b = \sum_{l=1}^{L} b_l$ where each b_l is a vector of length p with exactly one non-zero element; see susie and Wang et al (2020) for details.

In practice, this is accomplished by calling susie_suff_stat with XtX = R and Xty = z, and fixing residual_variance = 1. (Values for n and yty are also required by susie_suff_stat. They do not affect inference when the residual variance is fixed, but they do affect the interpretation of scaled_prior_variance; we set n=2,yty=1 so that $var(y) = yty/(n-1) = 1$.) Additional arguments to be passed to susie_suff_stat can be provided via ....

Value

A "susie" object with the following elements:

alpha	An L by p matrix of posterior inclusion probabilites.
mu	An L by p matrix of posterior means, conditional on inclusion.
mu2	An L by p matrix of posterior second moments, conditional on inclusion.
lbf	log-Bayes Factor for each single effect.
lbf_variable	log-Bayes Factor for each variable and single effect.
V	Prior variance of the non-zero elements of b.
elbo	The value of the variational lower bound, or "ELBO" (objective function to be maximized), achieved at each iteration of the IBSS fitting procedure.
Rr	A vector of length p, equal to R %*% colSums(alpha*mu).
sets	Credible sets estimated from model fit; see susie_get_cs for details.
pip	A vector of length p giving the (marginal) posterior inclusion probabilities for all p covariates.
niter	Number of IBSS iterations that were performed.
converged	TRUE or FALSE indicating whether the IBSS converged to a solution within the chosen tolerance level.

References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* **82** doi: 10.1101/501114.

X. Zhu and M. Stephens (2017). Bayesian large-scale multiple regression with summary statistics from genome-wide association studies. *Annals of Applied Statistics* **11**, 1561-1592.

Examples

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))

input_ss = compute_suff_stat(X,y,standardize = TRUE)
ss   = univariate_regression(X,y)
R    = with(input_ss,cov2cor(XtX))
zhat = with(ss,betahat/sebetahat)
res  = susie_rss(zhat,R,L = 10)
```

`susie_trendfilter` *Apply susie to trend filtering (especially changepoint problems), a type of non-parametric regression.*

Description

Fits the non-parametric Gaussian regression model $y = mu + e$, where the mean mu is modelled as $mu = Xb$, X is a matrix with columns containing an appropriate basis, and b is vector with a (sparse) SuSiE prior. In particular, when `order = 0`, the jth column of X is a vector with the first j elements equal to zero, and the remaining elements equal to 1, so that b_j corresponds to the change in the mean of y between indices j and j+1. For background on trend filtering, see Tibshirani (2014). See also the "Trend filtering" vignette, `vignette("trend_filtering")`.

Usage

```
susie_trendfilter(y, order = 0, standardize = FALSE, use_mad = TRUE, ...)
```

Arguments

`y`	An n-vector of observations ordered in time or space (assumed to be equally spaced).

order	An integer specifying the order of trend filtering. The default, order = 0, corresponds to "changepoint" problems (*i.e.*, piecewise constant mu). Although order > 0 is implemented, we do not recommend its use; in practice, we have found problems with convergence of the algorithm to poor local optima, producing unreliable inferences.
standardize	Logical indicating whether to standardize the X variables ("basis functions"); standardize = FALSE is recommended as these basis functions already have a natural scale.
use_mad	Logical indicating whether to use the "median absolute deviation" (MAD) method to the estimate residual variance. If use_mad = TRUE, susie is run twice, first by fixing the residual variance to the MAD value, then a second time, initialized to the first fit, but with residual variance estimated the usual way (by maximizing the ELBO). We have found this strategy typically improves reliability of the results by reducing a tendency to converge to poor local optima of the ELBO.
...	Other arguments passed to susie.

Details

This implementation exploits the special structure of X, which means that the matrix-vector product $X^T y$ is fast to compute; in particular, the computation time is $O(n)$ rather than $O(n^2)$ if X were formed explicitly. For implementation details, see the "Implementation of SuSiE trend filtering" vignette by running vignette("trendfiltering_derivations").

Value

A "susie" fit; see susie for details.

References

R. J. Tibshirani (2014). Adaptive piecewise polynomial estimation via trend filtering. *Annals of Statistics* **42**, 285-323.

Examples

```
set.seed(1)
mu = c(rep(0,50),rep(1,50),rep(3,50),rep(-2,50),rep(0,200))
y = mu + rnorm(400)
s = susie_trendfilter(y)
plot(y)
lines(mu,col = 1,lwd = 3)
lines(predict(s),col = 2,lwd = 2)

# Calculate credible sets (indices of y that occur just before
# changepoints).
susie_get_cs(s)

# Plot with credible sets for changepoints.
susie_plot_changepoint(s,y)
```

univariate_regression *Perform Univariate Linear Regression Separately for Columns of X*

Description

This function performs the univariate linear regression y ~ x separately for each column x of X. Each regression is implemented using `.lm.fit()`. The estimated effect size and stardard error for each variable are outputted.

Usage

```
univariate_regression(
  X,
  y,
  Z = NULL,
  center = TRUE,
  scale = FALSE,
  return_residuals = FALSE
)
```

Arguments

`X`	n by p matrix of regressors.
`y`	n-vector of response variables.
`Z`	Optional n by k matrix of covariates to be included in all regresions. If Z is not `NULL`, the linear effects of covariates are removed from y first, and the resulting residuals are used in place of y.
`center`	If `center = TRUE`, center X, y and Z.
`scale`	If `scale = TRUE`, scale X, y and Z.
`return_residuals`	Whether or not to output the residuals if Z is not `NULL`.

Value

A list with two vectors containing the least-squares estimates of the coefficients (`betahat`) and their standard errors (`sebetahat`). Optionally, and only when a matrix of covariates `Z` is provided, a third vector `residuals` containing the residuals is returned.

Examples

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
```

```
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = univariate_regression(X,y)
plot(res$betahat/res$sebetahat)
```

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